STANFORD UNIVERSITY
CS 224M, Fall 2009
Practice Questions

Questions	Points
1 True or False	/12
2 Short Answers	/12
3 Extensive-Form Games	/16
4 Game Theory	/25
5 VCG	/15
Total	/80

Name of Student: ________________________________

SUID: ____

The Stanford University Honor Code:

I attest that I have not given or received aid in this examination, and that I have done my share and taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honor Code.

Signed: ____

1. **[20 points] True or False**

   In the following, you are asked to decide if the statement is true or false. Each correct answer will get you 2 points, whereas an incorrect answer will *lose* you 2 points. (No penalty for blanks)

	Statement	True	False
(a)	For any extensive-form game, there is a unique subgame perfect Nash equilibrum.		
(b)	The pair of strategies (always defect, always defect) is a Nash equilibrium in the infinitely repeated prisoner's dilemma under discounted rewards.		
Note:	**There will be a few more T or F questions on the real midterm.**		

2. **[12 points] Short Answers**

   For these questions, you do *not* need to show how you derive the answer for full credit. However, if your answer is incorrect, you may get partial credit if you have shown your derivation.

   (a) [4 points] Consider the following game:

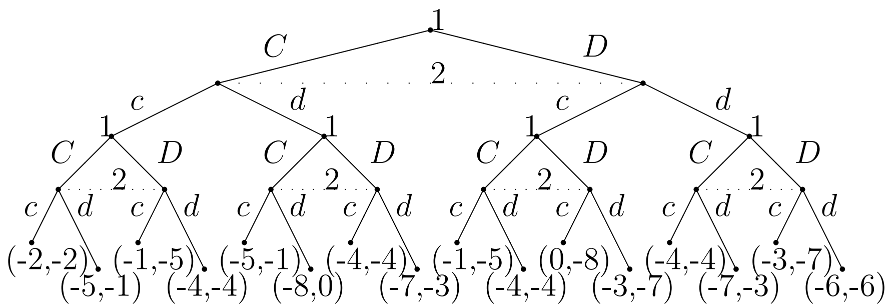


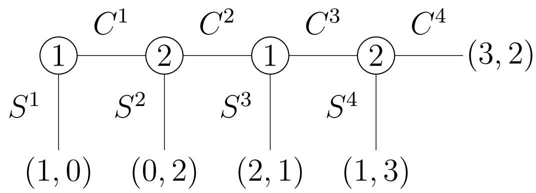
Figure 1: Twice-played Prisoner's Dilemma in extensive form.

   How many (pure) strategy profiles are there?

(b) [4 points] Suppose you are given a solver that finds a Nash equilibrium for any three-player *zero-sum* game. Describe how you can use it to find a Nash equilibirum for an arbitrary two-player game (not necessarily zero-sum).

3. **[16 points] Extensive-Form Games**

   Consider the following centipede game.



   Suppose this game will be played repeatedly. For notations, for the strategy X in a stage game, we will denote it being played in round 2 by X_2. For example, if agent 1 chooses to play S_1 in round 2, we will denote this by S^1_2.

   (a) [3 points] Suppose the game is played repeatedly three times. Is the pair of strategies $((C^1_1, C^3_1, C^1_2, C^3_2, C^1_3, C^3_3), (C^2_1, C^4_1, C^2_2, C^4_2, C^2_3, C^4_3))$ in a Nash equilibrium? Why or why not?

   (b) [3 points] Suppose the game is played repeatedly three times. Is the pair of strategies $((S^1_1, S^3_1, S^1_2, S^3_2, S^1_3, S^3_3), (S^2_1, S^4_1, S^2_2, S^4_2, S^2_3, S^4_3))$ in a Nash equilibrium? Why or why not?

(c) [10 points] Suppose the game is played repeatedly *forever*. Let the utilities of the agents be measured by discounted payoffs with discount factor γ. In other words, if the payoff to agent 1 using strategy $\mathcal{S}$ in round i is p_i, then

$$u_1(\mathcal{S}) = \sum_{t=1}^{\infty} \gamma^t p_t$$

Describe a strategy such that if both players play this strategy, they are in equilibrium (for the repeated game); and for each round, they play $((C^1, C^3), (C^2, C^4))$. What's the minimum γ for this strategy to be in equilibrium?

4. **[22 points] (Social Choice Functions)** Consider the instant runoff voting (IRV) scheme discussed in class. Under this social choice function, each voter submits a strict total ordering over the candidates, and the winning candidate is determined as follows. For each candidate, count the number of times that it appears at the top of a voter's ordering. If a candidate is ranked first by more than one half of the voters, then it is declared the winner. Otherwise, remove the candidate with the least number of first place votes from each voter's ordering (if there is a tie, remove the candidate among those tied that appears first alphabetically). Repeat this process until a candidate has a majority of the first place votes.

   (a) Is IRV is incentive compatible (that is – is it a dominant strategy for each agent to declare his preference ordering truthfully)? If so, informally justify why this is true. Otherwise, present a counterexample.

   (b) Recall from the course reader the *Condorcet condition*, which requires that if a candidate x is chosen, then for any candidate $y \neq x$ it must be the case that at least half the voters prefer x to y. Does IRV satisfy this condition? If so, informally justify why it does. Otherwise, present a counterexample.

5. **[15 points] (Combinatorial Auctions)**

   Recall the definitions of bidding languages from the course reader. An OR bid says that the bidder can accept any number of constituent bids. An XOR bid says that the bidder is willing to accept at most one constituent bid.

   The *OR-of-XOR* bidding language is a language where all bids are of the form $X_1 \vee \ldots \vee X_n$, where each X_i is of the form $A_1 \oplus \ldots \oplus A_{n_i}$, with each A_i being an atomic bid.

   The *XOR-of-OR* bidding language has opposite form: each bid looks like $O_1 \oplus \ldots \oplus O_n$, where each O_i is of the form $A_1 \vee \ldots \vee A_{n_i}$, with each A_i being an atomic bid.

   Finally, *OR** language is an *OR* language augmented with *dummy* goods.

   (a) [6 points] Prove theorem 11.3.8: Any valuation that can be represented by an *OR-of-XOR* bids with s atomic bids can also be represented by *OR** bids of s atomic bids, using at most s dummy goods.

(b) [9 points] Prove theorem 11.3.9: Any valuation that can be represented by an *XOR-of-OR* bids with s atomic bids can also be represented by *OR** bids with s atomic, using at most s^2 dummy goods.

(c) [**Extra Credit: 2 points**] Show how to construct an *OR-of-XOR* bid of a given size s such that in fact $\Omega(s)$ dummy goods must be used to represent it in *OR** language.

(d) [**Extra Credit: 3 points**] Show how to construct an *XOR-of-OR* bid of a given size s that indeed requires $\Omega(s^2)$ dummy goods to represent it in an *OR** language (i.e. it requires at least $c \cdot s^2$ dummy goods for some constant c).