CS224M, Fall 2009-10
Homework #4 : Mechanisms & Auctions
Handout #6

Note: Homework #4 is due on **November 16, 2009** in class. This problem set carries *less* weight than the previous assignments, and the maximum score for this assignment is 75. With this shorter assignment, we hope you will be able to find more time to prepare for the midterm and the final paper proposal.

1. **[15 points] (VCG Auction for Networks)**

   Consider the transportation network shown in Figure 1. Instead of selling goods to the agents, in this setting the agents (A through F) are selling (or, renting out) routes to the auctioneer. Each directed link is owned by a separate agent, and the valuation for agent i for his link being used is v_i. This value is always negative, to denote the fact that it is a cost. An agent has a value of 0 for his link remaining unused. We will run a VCG mechanism, in which an outcome $x \in X$ consists of a selection of links that form a path from S to T (there are 4 such paths in this network). Determine the route selected and the payments made to each of the agents when we run the VCG mechanism.

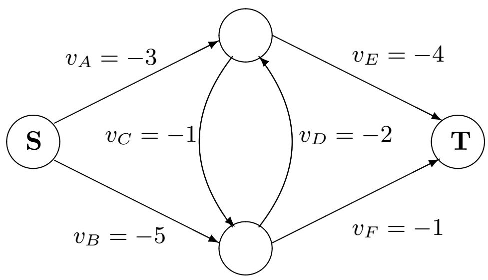

Figure 1: A transportation network with self-interested agents.

2. **[30 points] (Budget Balance and Mechanism Design)**

   It is easy to construct examples where the VCG mechanism is not budget balanced. While budget balance is not well-motivated in auctions, it is important for mechanisms in other domains (e.g., a public project). But it seems like an easy enough problem to solve– we can just evenly redistribute any money that was collected by the mechanism (or, tax all agents equally if the net payment to the agents was positive). Below are two attempts to create a proposed, budget-balanced version of the VCG Mechanism (even though Vickrey, Clarke, and Groves would certainly want any reference to them removed from the name).

   (a) [15 points] On the first attempt, we will convert what was p_i into a temporary variable t_i. Then, the new payments p_i contain an equal redistribution of the sum of the original payments.

   The *Budget-Balanced VCG Mechanism* is a direct mechanism $M(\hat{v}) = (x, p_1, \ldots, p_n)$, where

- $x = \arg\max_{x \in X} \sum_{i \in N} \hat{v}_i(x)$, and
- $t_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(x(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(x(\hat{v}))$
- $p_i = t_i - \frac{1}{n} \sum_i t_i$

Unfortunately, there does not exist a dominant strategy incentive-compatible mechanism that is efficient and budget-balanced (Theorem 10.4.12). Show that this mechanism is no longer incentive-compatible.
(Hint: You can give an example in which the mechanism is not incentive-compatible)

(b) [15 points] Fortunately, all is not lost. By sacrificing certainty we can try to make it budget-balanced on *expectation*. The new payments p_i now contain an equal redistribution of the sum of expected payments for each agent.

Assume that bidders valuations v_i are randomly drawn from some joint commonly known distribution. Consider the following mechanism:

Expected Budget-Balanced VCG Mechanism is a direct mechanism $M(\hat{v}) = (x, p_1, \ldots, p_n)$, where

- $x(\hat{v}) = \arg\max_{x' \in X} \sum_{i \in N} \hat{v}_i(x')$, and
- $t_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(x(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(x(\hat{v}))$
- $p_i = t_i(\hat{v}) - \frac{1}{n} \sum_j E_v[t_j(v)]$

i. [5 points] Prove that truth-telling is still dominant in this new mechanism.

ii. [10 points] Show that this mechanism is budged-balanced on expectation (i.e. expected total payment by all agents is zero).

3. **[30 points] (Auctions and Revenue)**

Dan is auctioning an object that Alice, Bob and Charlie want to acquire. They all know that the value of the object to each of the three is *uniformly* and *independently* distributed between 0 and 100. That is, the value of the object to Alice is V_a, to Bob is V_b and to Charlie is V_c, where V_a, V_b, V_c are independent random variables uniformly distributed between 0 and 100. Each one knows its own value. Alice gets a payoff of 0 if she does not acquire the object and a payoff of $V_a - p$ if she does, where p is the clearing price of the auction. Payoffs to Charlie and Bob are similarly defined. Throughout the question, assume that all players are *risk-neutral.*

(a) [10 points] Suppose Dan conducts a first price sealed bid auction. Further, suppose all three players, Alice, Bob, and Charlie chooses the same strategy such that the strategies are in equilibrium. What price should Alice write on her sealed bid? (Note that this price can depend on V_a, which Alice knows, but not on V_b or V_c.)

(b) [5 points] What price should she bid if Dan conducts a second price sealed bid auction?

(c) [5 points] For this part, suppose $V_a = 80$. If Dan conducts a Dutch auction, at what price should Alice jump in and stop the auction?

(d) [10 points] Under each of the above auction types, how much should Dan expect to obtain for the object?