CS224M, Fall 2009-10
Homework #2 :
Handout #2

Note: Homework #2 is due on **October 19, 2009** in class

1. **[25 points] (Repeated Games)** Consider the 2-player normal form game in Table 1.

	L	R
A	$(0,0)$	$(3,1)$
B	$(0,0)$	$(3,1)$
C	$(1,4)$	$(1,4)$
D	$(2,5)$	$(2,5)$

Table 1: A two-player normal form game.

   (a) [5 points] List all of the pure strategy Nash equilibrium strategy profiles of this game.

   (b) [5 points] Now consider the discounted, infinitely repeated version of the game, in which each agent's discount factor is $\beta = 0.01$. Thus, if we denote by r_j the payoff that an agent receives in iteration j of the repeated game, then the agent's utility is $\sum_{j=1}^{\infty}(0.01)^j * r_j$. In each iteration of the repeated game, each agent remembers the history, which consists of all actions taken in previous iterations.

   Find a Nash equilibrium strategy profile of this repeated game. Briefly justify why your strategy profile is an equilibrium. You can describe each agent's strategy using words, but make sure that it is well-defined for all possible histories.

   (c) [5 points] Now set $\beta = 0.9$. Does there exist a Nash equilibrium of this repeated game which gives rise to the action profile (C, L) being played in each round? If so, state the equilibrium strategy profile and briefly justify why it is an equilibrium. If not, then briefly explain why it is impossible for such an equilibrium to exist.

   (d) [5 points] The redundancy of the normal form game in Table 1 suggests that it was constructed from an extensive form game. Find a *perfect information* extensive form game with only four leaves which is equivalent to the normal form game in Table 1. Write out the mapping of pure strategies in the extensive form game to actions in the normal form game.

   (e) [5 points] List all of the subgame perfect equilibrium strategy profiles of the extensive form game you found in the previous part.

2. **[25 points] (Bayesian Games)** Consider the following variation to the Rock, Paper, Scissors game. Suppose that, with probability a, player 1 faces a rational opponent (that believes there to be common knowledge of rationality) and, with probability $1 - a$, she faces an opponent that will play P for sure. That is, before the game, Nature select player 2's type and player 1 does not see Nature's choice.

   (a) [6 points] Draw the extensive form of this game.

   (b) [5 points] Denote strategies for player 2 as (P,P), (P,R), (P,S). Complete the following normal form representation of the game: (Since this is a zero-sum game, simply write player 1's expected payoff in that situation in each cell.)

	P,P	P,R	P,S
P		a	
R			
S			1-a

(c) [7 points] Find the equilibria of this game for $a = 1/3$.

(d) [7 points] Find the equilibria of this game for $a = 1/2$.

3. **[25 points] (Congestion Games)**

Many settings are naturally modeled as a congestion game, including traffic, computer networks, and even the decision of whether to attend a bar. In this problem we will consider a variant of the well-studied Santa Fe (or, El Farol) Bar Problem (mentioned on pg 170 of course reader), in which a set of people are simultaneously deciding whether or not to attend a bar. We'll consider a version of the problem where two students are simultaneously deciding between going to the bar and staying home. Each student has a preference for either going to a bar or staying at home. The students don't know each other's preferences, but know that they are drawn from a commonly known joint distribution. This distribution is described in Table 2. Starting from a baseline utility of zero, a student gains 1 unit of utility if he goes to the place that he prefers. However, the bar is (extremely) small, and thus both students lose 2 units of utility if they both attend the bar (independent of whether they gained 1 unit of utility based on their preference). Thus, for example, if they both prefer bar, and they both go to the bar, they each get a utility of $0 + 1 - 2 = -1$.

Possible World w_j	Student 1	Student 2	Common Prior $\mathcal{P}(w_j)$
w_1	b_1	b_2	0.15
w_2	b_1	$\neg b_2$	0.45
w_3	$\neg b_1$	b_2	0.15
w_4	$\neg b_1$	$\neg b_2$	0.25

Table 2: The common prior joint distribution on student preferences. b_i means that student i prefers to go to the bar, $\neg b_i$ means he prefers to stay home.

(a) [10 points] Model the setting as a Bayesian game (N, G, P, I) as in Definition 6.3.1 given on page 161 of the course reader. $N = \{1, 2\}$ is the set of agents, G is the set of games, P is the common prior over games, and $I = \{I_1, I_2\}$ are the partitions over games for the two agents. Your entire answer can be a figure similar to Figure 6.7, which shows the games, the common prior, and the partitions of the agents. Denote by B and H the actions of going to the bar and staying home, respectively.

(b) [5 points] Find a Bayes-Nash equilibrium of this game. You will justify your answer in part (c).

	BB	BH	HH	HB
BB	(-1.4,-1.7)		(0.6,0.7)	
BH		(0.7,0.7)		(0.1,-0.9)
HH	(0.4,0.3)		(0.4,0.7)	(0.4,0.0)
HB		(-0.3,0.7)		(-0.5,-0.5)

Table 3: Induced Normal Form for Problem 3(c).

(c) [10 points] Draw the payoff matrix of the induced normal form of this Bayesian Game and fill in enough of the payoffs to justify your equilibrium from part (b). We have given some entries in Table 3.

4. **[25 points] (Coalitional Games)**

   In this (fictional!) story, the instructor in CS224M allows the students to collaborate and write up together a particular problem in the homework assignment; points earned by a collaborating team are divided among the students in any way they agree on. There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate. The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score m points.

   (a) [3 points] Set this up as a coalitional game. Write it down in characteristic form.

   (b) [3 points] For what values of m is the game super-additive?

   (c) [4 points] Prove that for all $m < 6$ the core is empty.

   (d) [2 points] In the context of the current example, what does it mean in common sense terms (and in one short paragraph at most) that the core is empty?

   (e) [5 points] For $m = 6$, what does the core consist of? Prove your answer.

   (f) [3 points] For $m = 6$, what is the Shapley value of each player? Show how you derive the answer.

   (g) [5 points] For general m, what is the Shapley value of each player and why?

5. **[Food For Thought. 0 points] (More Centipede)**

   We highly recommend that you do this experiment, as it can be quite fun and educational. Recall the original centipede game on page 124 of the book. Team up with a classmate and play this game repeatedly, taking payoffs as seriously as you can. Try not to discuss the game in advance. Repeat the same experiment with somebody who hasn't had any exposure to game theory. Compare these experiences to each other and theoretical predictions.