CS224M, Fall 2009-10
Homework #1 : Game Theory
Handout #1

Note: Homework #1 is due on **October 05, 2009** in class

1. **[25 points] (2-Player Normal Form Game)**

   Consider the following game:

		Player 2		
		L	C	R
Player 1	T	8,8	5,6	-12,2
	M	0,8	8,2	-15,8
	B	2,-12	8,-9	4,4

   Where the first number in each square is the payoff of player 1 and the second number is player 2's payoff.

   (a) [5 points] Find all Pareto optimal pure strategy profiles.

   (b) [5 points] Find the pure strategy Nash Equilibria.

   (c) [15 points] Find all mixed-strategy Nash Equilibria. (Hint: look at the procedure on pages 62-63 of the book).

2. **[25 points] (3-Player Normal Form Game)**

   Consider the *3-player* normal form game in Table 1. Here each player has two strategies: (T,B) for player 1, (L,R) for player 2 and (N,F) for player 3. In this notation, player 3 gets to select left or right table, player 2 selects the column, and player 1 selects the row. For example, if they play (T, L, F) they each get $(2, 2, 6)$ respectively.

	L	R
T	$(5, 5, 5)$	$(2, 6, 2)$
B	$(6, 2, 2)$	$(3, 3, -1)$
N		

	L	R
T	$(2, 2, 6)$	$(-1, 3, 3)$
B	$(3, -1, 3)$	$(0, 0, 0)$
F		

   Table 1: A three-player normal form game.

   (a) [5 points] List all of the pure strategy Nash equilibrium profiles of this game.

   (b) [5 points] List all Pareto-optimal outcomes of this game

   (c) [5 points] Compute the maxmin value (or security level) for player 1.

   (d) [10 points] Is there a correlated equilibrium in which all of the players achieve payoffs strictly better than in the Nash equilibria you found in (a)? If yes, give an example, otherwise explain why not.

3. **[25 points] (Extensive-form Games: Perfect Information Game)**

   Consider a variation of the centipede game discussed on page 124 of the book. The play still alternates between the two players. They each start with a balance of \$0. On each player's turn she has the option to continue the game or stop the game. If she stops the other is allowed a final choice between accepting the accumulated payoff or discarding that payoff and replacing it by a balance of $\$-10$ for each player. If the player chooses to continue her balance is debited one dollar, and the opponent is credited with three dollars. If the second player has continued for a third time the game terminates; note that in this case each player will each receive \$6.

(a) [10 points] Draw the extensive form for this game.

(b) [10 points] How many Nash equilibria are there for this game if it only lasts for 3 rounds?

(c) [5 points] List the subgame perfect Nash equilibria in the original game.

4. **[25 points] (Extensive-form Games: Imperfect Information Game)**

Each part of this problem will use the two-player game of imperfect information given in Figure 1. However, the meaning of the numbers at the leaves will differ. In part (a), we consider a common-value game. Thus, the value at a leaf defines the payoff of both players. In parts (b) and (c), we switch to a zero-sum game. In that case, the value of a leaf defines the payoff of player 1, and the negative of the payoff of player 2. In each part, briefly justify your answer.

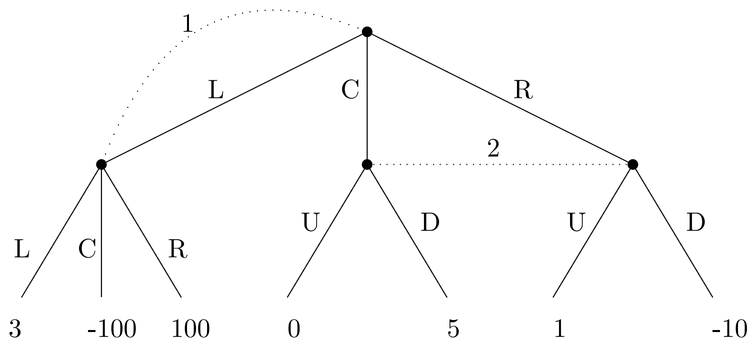

Figure 1: An imperfect information game in which each player has a single information set.

(a) [5 points] For the *common-value* game defined by Figure 1, list all *pure* strategy Nash equilibria ("none exists" is a possible answer).

(b) [5 points] For the *zero-sum* game defined by Figure 1, list all *pure* strategy Nash equilibria ("none exists" is a possible answer).

(c) [15 points] Now we will allow mixed (but not behavioral) strategies. For the *zero-sum* game defined by Figure 1, list all (possibly mixed) Nash equilibria. As there could be an infinite number of mixed Nash equilibria, you should use variables and give ranges over which strategy profiles constitute Nash equilibria

[Hints: (1) the characterization is simple. (2) use the fact that the game is a zero-sum game]