Introduction to Robotics (CS223A) Handout #1

(Winter 2011/12)

Review 1 Solutions

Date **January 25, 26**

(*D-H Parameters*) Consider the PRRR manipulator shown below:

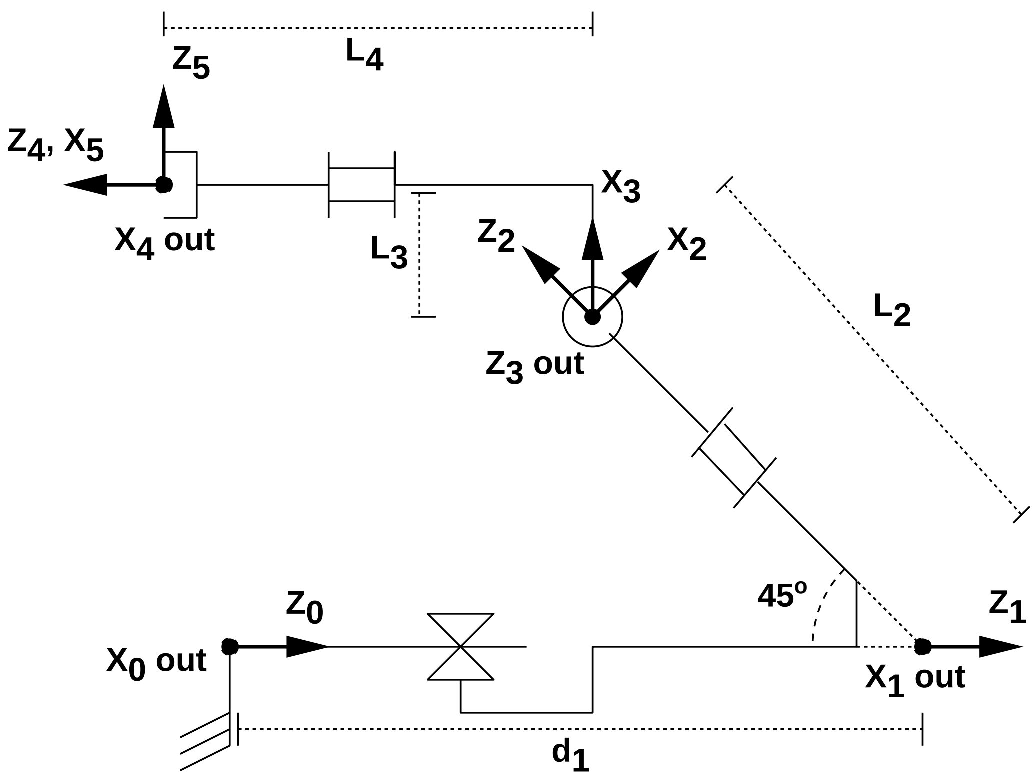


Figure 1: Schematic of an PRRP manipulator.

i	α_{i-1}	a_{i-1}	θ_i	d_i	conf. shown
1	$0°$	0	$0°$	d_1	$d_1 = d_1$
2	$135°$	0	θ_2	L_2	$\theta_2 = 90°$
3	$90°$	0	θ_3	0	$\theta_3 = 45°$
4	$-90°$	L_3	θ_4	L_4	$\theta_4 = -90°$
5	$-90°$	0	$-90°$	0	n/a

(*Kinematics*) Examine the following RPP manipulator:

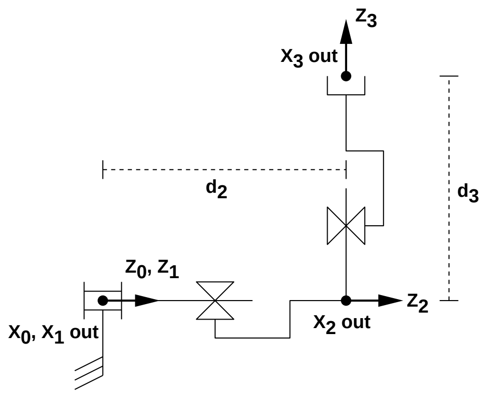


Figure 2: Schematic of an RPP manipulator.

1. Given the transformation matrix

$$ {}^1_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

find the forward kinematics matrix ${}^0_3T$.

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^0_3T = {}^0_1T\,{}^1_3T = \begin{bmatrix} c_1 & 0 & s_1 & d_3 s_1 \\ s_1 & 0 & -c_1 & -d_3 c_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2. For the joint angles $\theta_1 = 30°$, $d_2 = 4$, and $d_3 = 6$, what is the location of the end-effector (in Frame $\{0\}$)?

$$ {}^0p_E = \begin{bmatrix} d_3 s_1 \\ -d_3 c_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -3\sqrt{3} \\ 4 \end{bmatrix} $$

3. In contrast to part (b), suppose we know the position of the end-effector:

$$ {}^0p_E = \begin{bmatrix} 2\sqrt{2} \\ 2\sqrt{2} \\ 5 \end{bmatrix} $$

What are the values of the joint angles θ_1, d_2 and d_3 for this position? (Assume $d_3 > 0$.)

$$ \left\{ \begin{array}{rcl} 2\sqrt{2} & = & d_3 s_1 \\ 2\sqrt{2} & = & -d_3 c_1 \\ 5 & = & d_2 \end{array} \right\} \qquad \begin{array}{rcl} d_3^2 & = & 16 \\ d_3 & = & 4 \\ & & \\ s_1 & = & \frac{1}{\sqrt{2}} \\ c_1 & = & -\frac{1}{\sqrt{2}} \\ \theta_1 & = & 135° \end{array} $$

(*Rotations*) Given the following 3x3 matrix,

$$R = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}.$$

1. Show that it is a rotation matrix.
2. Determine a unit vector that defines this axis of rotation and the angle (in degrees) of rotation.
3. What are the Euler parameters $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ of R?

For (a), show that each column has magnitude of 1, and the columns are orthogonal (dot product = 0). Alternatively, you can show that the transpose is the inverse. There is an additional condition : the determinant is +1.

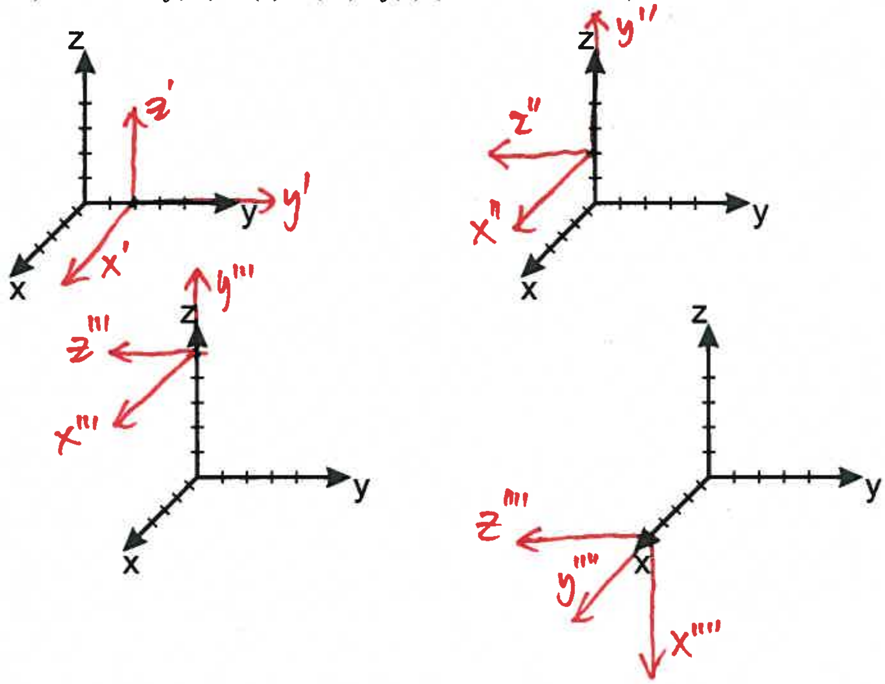
For (b), we want the screw axis ($\hat{K}$) and angle (θ). From the equations in the book,

$$\theta = \arccos(\tfrac{r_{11}+r_{22}+r_{33}-1}{2}) = \arccos(-1/2) = 120^\circ,$$

$$\hat{K} = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ \sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ \sqrt{2/3} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/3 \\ \sqrt{6}/3 \\ 0 \end{bmatrix}$$

For (c), we use the Equations for the Euler parameters; where $\theta/2 = 60^\circ$, and $\sin 60^\circ = \sqrt{3}/2$, $\cos 60^\circ = 1/2$, and $\varepsilon_1 = 1/2$, $\varepsilon_2 = \sqrt{2}/2$, $\varepsilon_3 = 0$, $\varepsilon_4 = 1/2$

4. (*Rotations*) $\mathbf{R}T_N = R_y(90)D_z(3)R_x(90)D_y(2)$ (from right to left)



$\mathbf{R}T_N = R_y(90)D_z(3)R_x(90)D_y(2)$ (from left to right)