Introduction to Robotics (CS223A) Homework #6

(Winter 2008/2009) Due: **Tuesday, March 10**

- Please include a little grid like this at the top of your first page of homework:

1	2	3	Total

1. For a certain RR manipulator, the equations of motion are given by

$$\begin{bmatrix} 4+c_2 & 1+c_2 \\ 1+c_2 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -s_2(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2) \\ s_2\dot{\theta}_1^2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

   (a) Assume that joint 2 is locked at some value θ_2 using brakes and joint 1 is controlled with a PD controller, $\tau_1 = -40\dot{\theta}_1 - 400(\theta_1 - \theta_{1d})$. What is the minimum and maximum inertia perceived at joint 1 as we vary θ_2? What are the corresponding closed-loop frequencies?

   (b) Still assuming that joint 2 is locked, at what values of θ_2 do the minimum and maximum damping ratios occur?

   (c) Now assume that both joints are free to move, and that this system is controlled by a partitioned PD controller, $\tau = \alpha\tau' + \beta$. Design a partitioned, trajectory-following controller (one that tracks a desired position, velocity and acceleration) which will provide a closed-loop frequency of 10 rad/sec on joint 1 and 20 rad/sec on joint 2 and be critically damped over the entire workspace. That is, let

$$\tau' = \ddot{\theta}_d - \begin{bmatrix} k'_{v_1} & 0 \\ 0 & k'_{v_2} \end{bmatrix} (\dot{\theta} - \dot{\theta}_d) - \begin{bmatrix} k'_{p_1} & 0 \\ 0 & k'_{p_2} \end{bmatrix} (\theta - \theta_d),$$

   then find the matrices α and β and the vector τ, along with the necessary gains k'_{v_i} and k'_{p_i}.

   (d) If $\theta_2 = 180°$, what is the steady-state error vector for a given disturbance torque, $\tau_{dist} = [2\ 4]^T$?

2. Consider the 1-DOF system described the equation of motion, $4\ddot{x} + 20\dot{x} + 25x = f$.

   (a) Find the natural frequency ω_n and the natural damping ratio ζ_n of the natural (passive) system ($f = 0$). What type of system is this (oscillatory, overdamped, etc.) ?

   (b) Design a PD controller that achieves critical damping with a closed-loop stiffness $k_{CL} = 36$. In other words, let $f = -k_v\dot{x} - k_p x$, and determine the gains k_v and k_p. Assume that the desired position is $x_d = 0$.

   (c) Assume that the friction model changes from linear ($20\dot{x}$) to Coulomb friction, $30 sign(\dot{x})$. Design a control system which uses a non-linear model-based portion with trajectory following to critically damp the system at all times and maintain a closed-loop stiffness of $k_{CL} = 36$. In other words, let $f = \alpha f' + \beta$ and $f' = \ddot{x}_d - k'_v(\dot{x} - \dot{x}_d) - k'_p(x - x_d)$. Then, find f,α,β,f',k'_p and k'_v. Note that f is an m-mass control, and f' is a unit-mass control. Use the definition of error, $e = x - x_d$.

(d) Given a disturbance force $f_{dist} = 4$, what is the steady-state ($\ddot{e} = \dot{e} = 0$) error of the system in part (c)?

3. Consider the 1-DOF system with equation of motion:

$$f = ml^2\ddot{\theta} + v\dot{\theta} + mlg\cos(\theta)$$

We are using a control strategy which compensates for the non-linear part of the system and has a unit-mass linear controller for trajectory tracking:

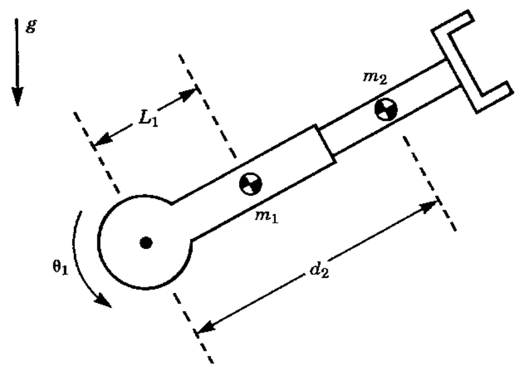
$$\begin{aligned} f &= \alpha f' + \beta \\ \alpha &= \hat{m}l^2 \\ \beta &= v\dot{\theta} + \hat{m}lg\cos(\theta) \\ f' &= \ddot{\theta}_d - k'_v(\dot{\theta} - \dot{\theta}_d) - k'_p(\theta - \theta_d) \end{aligned}$$

(where $\hat{m}$ is the estimate of the mass m of our system.)

If there is an error in our mass estimate, given by $\psi = m - \hat{m}$, then what is the resultant *steady-state* position error of the controlled system? Assume position error is given by $e = \theta - \theta_d$. Your answer should be in terms of ψ, $\hat{m}$, l, k'_p, $\ddot{\theta}_d$, θ, and g.

4. **Extra credit problem**

Consider the 2-link RP manipulator shown below:



Its equations of motion were derived in the Lecture Notes and are shown here:

$$\begin{aligned} \tau_1 &= (m_1L_1^2 + I_{zz1} + I_{zz2} + m_2d_2^2)\,\ddot{\theta}_1 + 2m_2d_2\dot{\theta}_1\dot{d}_2 + (m_1L_1 + m_2d_2)g\cos(\theta_1) \\ \tau_2 &= m_2\ddot{d}_2 - m_2d_2\dot{\theta}_1^{\,2} + m_2g\sin(\theta_1) \end{aligned}$$

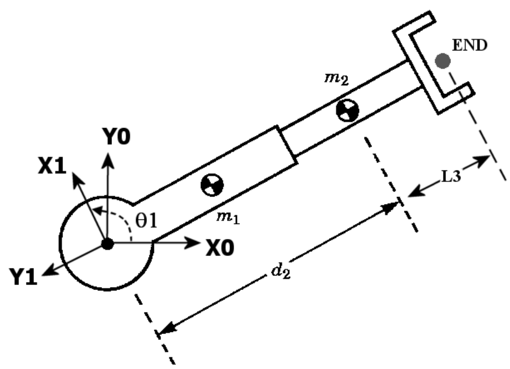
The manipulator parameters have the following numerical values: $L_1 = 0.2m$, $m_1 = 1.0kg$, $m_2 = 0.8kg$, $I_{zz1} = 0.1kgm^2$, $I_{zz2} = 0.07kgm^2$, and the range of d_2 is between $0.5m$ and $1.0m$.

(a) The system is controlled by a joint-space dynamic decoupling control, $\tau = \alpha\tau' + \beta$, which compensates the non-linear part of the system, decouples the dynamics, and tracks a desired trajectory (ie. position, velocity and acceleration) separately for each joint. Leaving only the feedback gains (k'_{p1}, k'_{p2}, k'_{v1}, k'_{v2}) as symbols, give values for the matrix α, vector β, and vectors τ and τ'. Note: you should also leave the joint variables (θ_1, d_2) and joint velocities ($\dot{\theta}_1$, $\dot{d}_2$) as symbols.

(b) Find the values for the gains k'_{p1}, k'_{p2}, k'_{v1}, k'_{v2} such that the closed-loop system for joint 1 is critically damped with natural frequency of 20 rad/sec, and the closed-loop system for joint 2 is critically damped with natural frequency of 25 rad/sec.

(c) Consider the original equations of motion (ie. without a controller), when $d_2 = 0.6m$. For joint 1, what is the effective inertia "seen" by the joint if we have gearing with ratio $\eta = 5$ and motor inertia $I_m = 0.004kgm^2$?

Consider again the original system (ie. no controller or gearing). You are given DH coordinate frames as shown below:



The length from the center of mass of link 2 to the end-effector is L_3. In this case, the end-effector position in the plane is:

$$\mathbf{0}P_{end} = \begin{bmatrix} s_1(d_2 + L_3) \\ -c_1(d_2 + L_3) \end{bmatrix}$$

(d) Use $\mathbf{0}P_{end}$ to compute the Jacobian for linear velocity at the end-effector in frame $\{0\}$.

(e) Using your answer from part (d), the configuration $\theta_1 = 45°$, $d_2 = 0.6m$, and assuming $L_3 = 0.2m$, compute the system's mass matrix M_x in frame $\{0\}$ when the dynamics are written in terms of the operational space coordinates.