Introduction to Robotics (CS223A) — Homework #4

(Winter 2009/2009) — Due: **Wednesday, February 18**

- Please include a little grid like this at the top of your first page of homework:

1	2	3	Total

1. The mobile robot shown below can be represented as a PPRR manipulator with 4 links, as shown in the schematic.

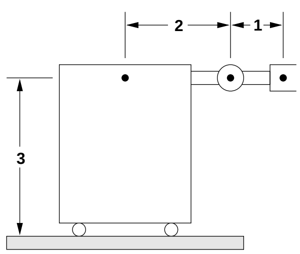

Figure 1: Mobile robot

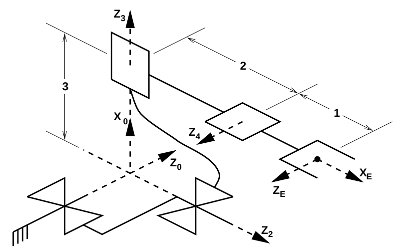

Figure 2: Mobile robot

Luckily, you do not need to compute the forward kinematics, because they are given to you here:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^0_2T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^0_3T = \begin{bmatrix} 0 & 0 & 1 & 3 \\ s_3 & c_3 & 0 & d_2 \\ -c_3 & s_3 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} s_4 & c_4 & 0 & 3 \\ s_3c_4 & -s_3s_4 & -c_3 & d_2 + 2s_3 \\ -c_3c_4 & c_3s_4 & -s_3 & d_1 - 2c_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^0_ET = \begin{bmatrix} s_4 & c_4 & 0 & 3 + s_4 \\ s_3c_4 & -s_3s_4 & -c_3 & d_2 + 2s_3 + s_3c_4 \\ -c_3c_4 & c_3s_4 & -s_3 & d_1 - 2c_3 - c_3c_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(a) Find the set of all values (q_1, q_2, q_3, q_4) such that the end-effector of the robot reaches the point ${}^0\mathbf{p} = (4, 2, 2)$. Describe this set in the form of the equations $f(q_1, q_2) = 0$, $q_3 = g(q_1, q_2)$, and $q_4 = c$, where c is some scalar. What does the curve $f(q_1, q_2) = 0$ represent?

(b) Derive the basic Jacobian, J_0, that expresses the linear and angular velocity of the end-effector as a function of the joint velocities.

(c) Assuming that the arm is straight out ($\theta_4 = 0°$), find the joint velocities that will achieve an end-effector angular velocity of $\omega = [1\ 0\ 0]^T$ while keeping the end-effector's position stationary. Describe this set by an equation of the form $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q})$. Now give a relationship $g(\dot{q}_1, \dot{q}_2) = 0$ that must hold for any configuration $\mathbf{q}$ of the robot which satisfies the problem so far. What does the curve $g(\dot{q}_1, \dot{q}_2) = 0$ represent?

2. Consider the following RRRR manipulator in Figure 3 (image courtesy J. J. Craig):

   It has the following forward kinematics and rotational Jacobian:

$$
{}^0_4T = \begin{bmatrix} c_{12}c_{34} - \frac{\sqrt{2}}{2}s_{12}s_{34} & -c_{12}s_{34} - \frac{\sqrt{2}}{2}s_{12}c_{34} & \frac{\sqrt{2}}{2}s_{12} & \sqrt{2}c_{12}c_3 - s_{12}(s_3 - 1) + c_1 \\ s_{12}c_{34} + \frac{\sqrt{2}}{2}c_{12}s_{34} & -s_{12}s_{34} + \frac{\sqrt{2}}{2}c_{12}c_{34} & \frac{\sqrt{2}}{2}c_{12} & \sqrt{2}s_{12}c_3 + c_{12}(s_3 - 1) + s_1 \\ \frac{\sqrt{2}}{2}s_{34} & \frac{\sqrt{2}}{2}c_{34} & \frac{\sqrt{2}}{2} & s_3 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0J_\omega = \begin{bmatrix} 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 \\ 1 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}
$$

   (a) Find the basic Jacobian J_o in the $\{0\}$ frame, for the position $\mathbf{q} = [0, 90^0, -90^0, 0]^T$. ($\mathbf{q}$ is the vector of joint variables.)

   (b) A general force vector is applied to the origin of frame $\{4\}$ and measured in frame $\{4\}$ to be $[0, 6, 0, 7, 0, 8]^T$. For the position in (a), determine the joint torques that statically balance it.

   (c) Consider the same configuration as above. A screw driver is gripped in the end-effector so that its tip is along $\hat{Z}_4$ at a distance of 9 units of length from the origin of frame $\{4\}$. What is the force and torque the screw driver tip applies when the same joint torques that were determined in part (b) are applied?

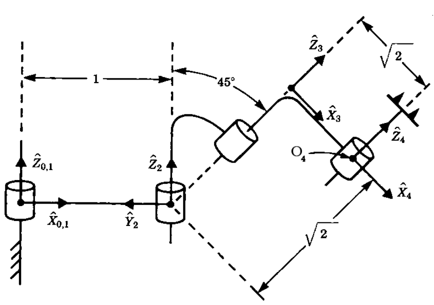

Figure 3: RRRR manipulator

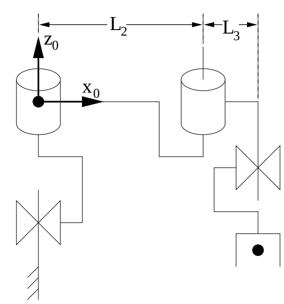

Figure 4: PRRP manipulator

3. Consider the PRRP manipulator schematic shown in Figure 4.

   (a) Assuming no joint limits, sketch the workspace of this manipulator. Be sure to include dimensions in your drawing. Assume $L_2 > L_3$.

   (b) Describe the (3D) dexterous workspace of this manipulator.

   (c) With no joint limits, if we are considering only the position of the end effector, how many inverse kinematic solutions are there (in general)? Explain briefly.

   (d) Imagine that we remove the first prismatic joint, so that the first revolute joint now rotates around the base. Repeat part (c) for such an RRP manipulator.

   (e) Imagine that we further modify the manipulator from part (d) by inserting another revolute joint between the two existing revolute joints, whose axis is oriented in the same direction as the other two. Repeat part (c) for such an RRRP manipulator.