

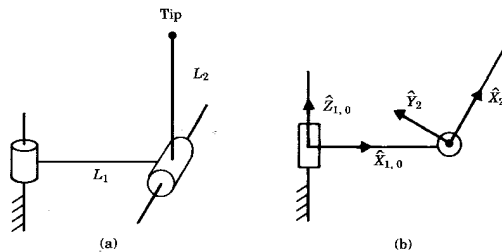
- Please include a little grid like this at the top of your first page of homework:

1	2	3	4	Total

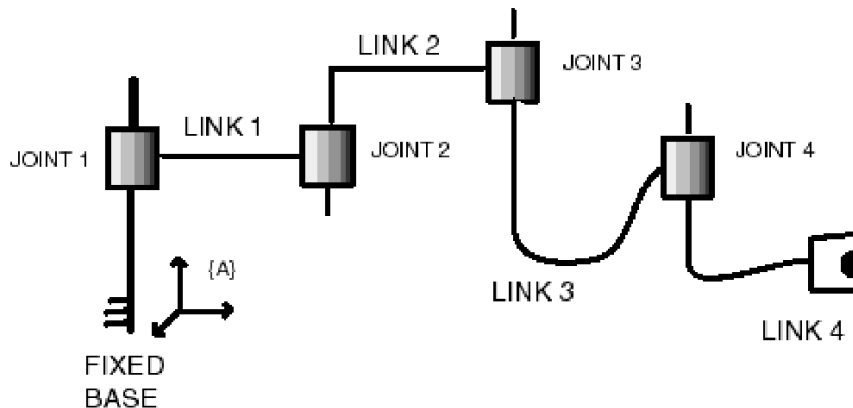
- Looking at equation 2.7 of the Lecture Notes, give a geometric interpretation of why  $t_{13} = 0$ . *Hint: Consider what the third column represents; your answer should be only one or two sentences.*
- For the 2-link manipulator shown, the link transformations  ${}^0_1T$  and  ${}^1_2T$  were determined. Their product is:

$${}^0_2T = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & l_1c_1 \\ s_1c_2 & -s_1s_2 & -c_1 & l_1s_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The frame assignments used are indicated below in the figure. Note that frame 0 is coincident with frame 1 when  $\theta_1$  is 0. The length of the second link is  $l_2$ . Find an expression for the vector  ${}^0P_{tip}$  which locates the tip of the arm relative to the 0 frame (figure courtesy of J. J. Craig).



- The following sketch represents a generic open, serial, kinematic-chain.



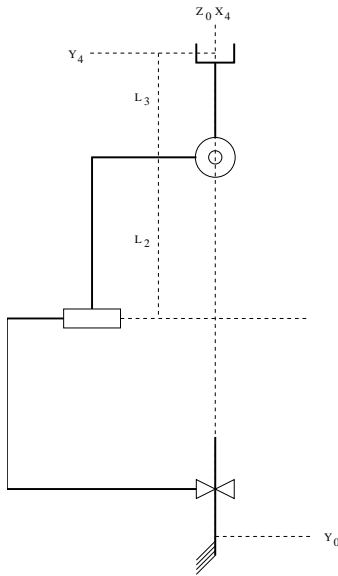
Here each kinematic joint connects two adjacent members. Assume that the relative displacement between adjacent members  $i - 1$  and  $i$  is described by an operator  $T_i$  that is a  $4 \times 4$  matrix whose elements are computed in a coordinate frame  $\{A\}$  fixed to the base of the chain. Now, if each member is displaced in sequence, *starting from the free end*, the displacement operator for the resultant total displacement of the free end will be given by  $T_1 T_2 T_3 T_4$ . (Note: In this problem you are to use only displacements operators, not coordinate transformations)

However, if the displacements are done in the reverse order, ie. *starting at the fixed end*, and moving in the sequence 1, 2, 3, 4, then the operators  $T_2$ ,  $T_3$ , and  $T_4$  no longer represent the actual displacements.

Determine, in terms of the original  $T_i$ :

- The operator for joint 2, when its displacement is done *after* the displacement in joint 1. Let us call this operator  $T'_2$
- The operator for joint 3 when its displacement follows the displacement in joints 1 and 2 (from part (a)). Let us call this operator  $T'_3$
- The operator for joint 4 when its displacement follows the displacement in joints 1, 2 and 3 (from part (b)). Let us call this operator  $T'_4$
- Using your results for parts (a), (b) and (c), show that the resulting displacement operator for the free end is still  $T_1 T_2 T_3 T_4$

4. Consider the PRR manipulator shown below.



$i$	$\alpha_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$	conf. shown
1					$d_1 = d_1$
2					$\theta_2 = -90^\circ$
3					$\theta_3 = 0^\circ$
4					n/a

- The configuration of the manipulator shown above is when  $d_1 = d_1$ ,  $\theta_2 = -90^\circ$ , and  $\theta_3 = 0^\circ$ . Assign the frames  $\{1\}$ ,  $\{2\}$ , and  $\{3\}$  consistent with the given configuration. Use the conventions studied in class.
- Introduce appropriate D-H parameters where necessary and label them on the figure.
- Fill in the table above.
- Derive the forward kinematics ( ${}^0_4T$ ) of the manipulator.
- What is the configuration of the manipulator when  ${}^0P_{4ORG} = [-L_2 \quad L_3 \quad d_1]^T$ ?
- Sketch the workspace of the manipulator. The joint limits are  $L_2 = 2L_3 = 0.5m$ ,  $0.5m \leq d_1 \leq 1.0m$ ,  $-180^\circ \leq \theta_2 \leq 0^\circ$ , and  $-90^\circ \leq \theta_3 \leq 0^\circ$ . Assume that the workspace is not limited by self-collisions.