

(Winter 2009/2009)

Due: Wednesday, January 21

Some tips for doing CS223A problem sets:

- Use abbreviations for trigonometric functions (e.g.  $c\theta$  for  $\cos(\theta)$ ,  $s_1$  or  $s\theta_1$  for  $\sin(\theta_1)$ ) in situations where it would be tedious to repeatedly write sin, cos, etc.
  - Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
  - If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.
1. A frame  $\{B\}$  and a frame  $\{A\}$  are initially coincident. Frame  $\{B\}$  is rotated about  $\hat{Y}_B$  by an angle  $\theta$ , and then rotated about the new  $\hat{Z}_B$  by an angle  $\phi$ . Determine the  $3 \times 3$  rotation matrix,  ${}^A_B R$ , which will transform the coordinates of a position vector from  ${}^B \mathbf{P}$ , its value in frame  $\{B\}$ , into  ${}^A \mathbf{P}$ , its value in frame  $\{A\}$ .
  2. A vector  ${}^A P$  is rotated about  $\hat{X}_A$  by  $\phi$  degrees and is subsequently rotated about  $\hat{Z}_A$  by  $\theta$  degrees.
    - (a) Give the rotation matrix which accomplishes these rotations in the given order.
    - (b) What is the result if  $\phi = 30^\circ$  and  $\theta = 45^\circ$ ?
  3. A vector is given by

$${}^B P = \begin{bmatrix} 1.0 \\ 5.0 \\ 10.0 \end{bmatrix}.$$

Given

$${}^B_A T = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 3.0 \\ 0.0 & 0.6 & 0.8 & -1.0 \\ 0.0 & -0.8 & 0.6 & 1.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix},$$

compute  ${}^A P$ .

4. Given the following  $3 \times 3$  matrix:

$$\begin{bmatrix} 0.7905 & -0.3864 & 0.4752 \\ 0.6046 & 0.3686 & -0.7061 \\ 0.0977 & 0.8455 & 0.5250 \end{bmatrix}$$

- (a) Verify that, within practical numerical limits, it does in fact represent a rigid body rotation or a rotational Cartesian coordinate transformation.
- (b) Determine a set of Euler parameters defined by this matrix.
- (c) Determine a unit vector describing the axis of rotation, and the angle (in degrees) of the rotation.