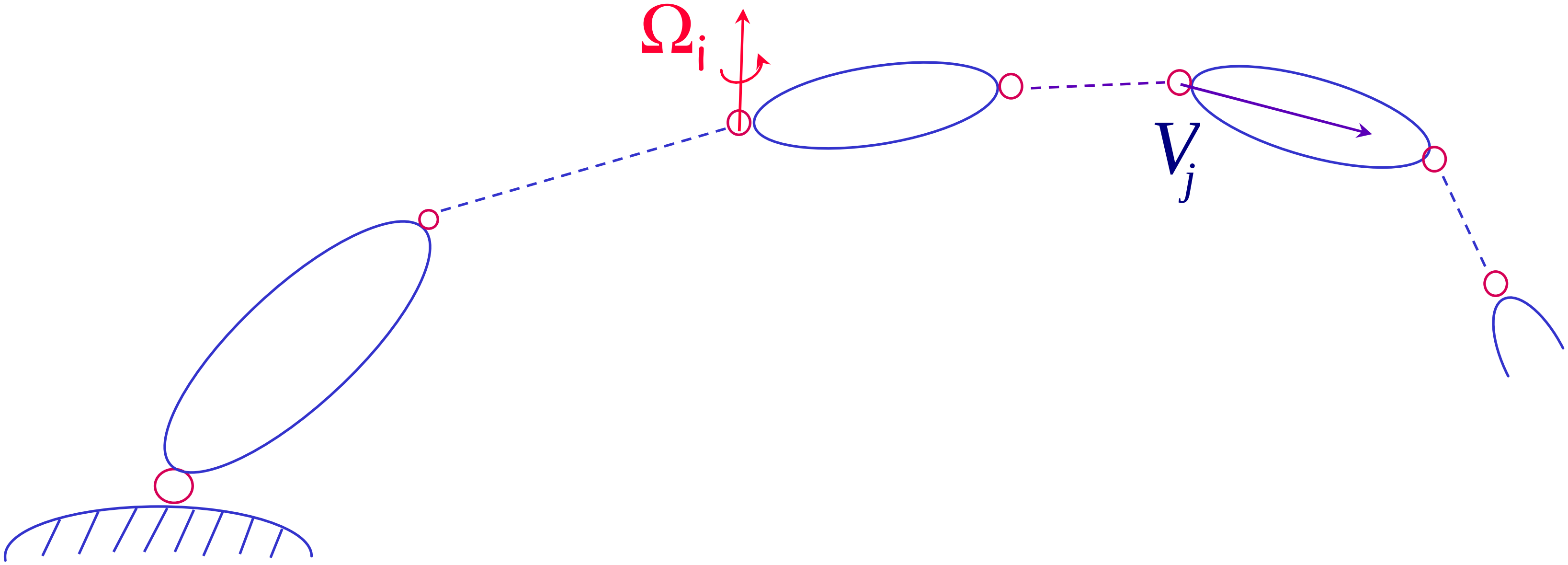
The Jacobian (EXPLICIT FORM)



Revolute Joint $\Omega_i = Z_i \dot{q}_i$

Prismatic Joint $V_i = Z_i \dot{q}_i$

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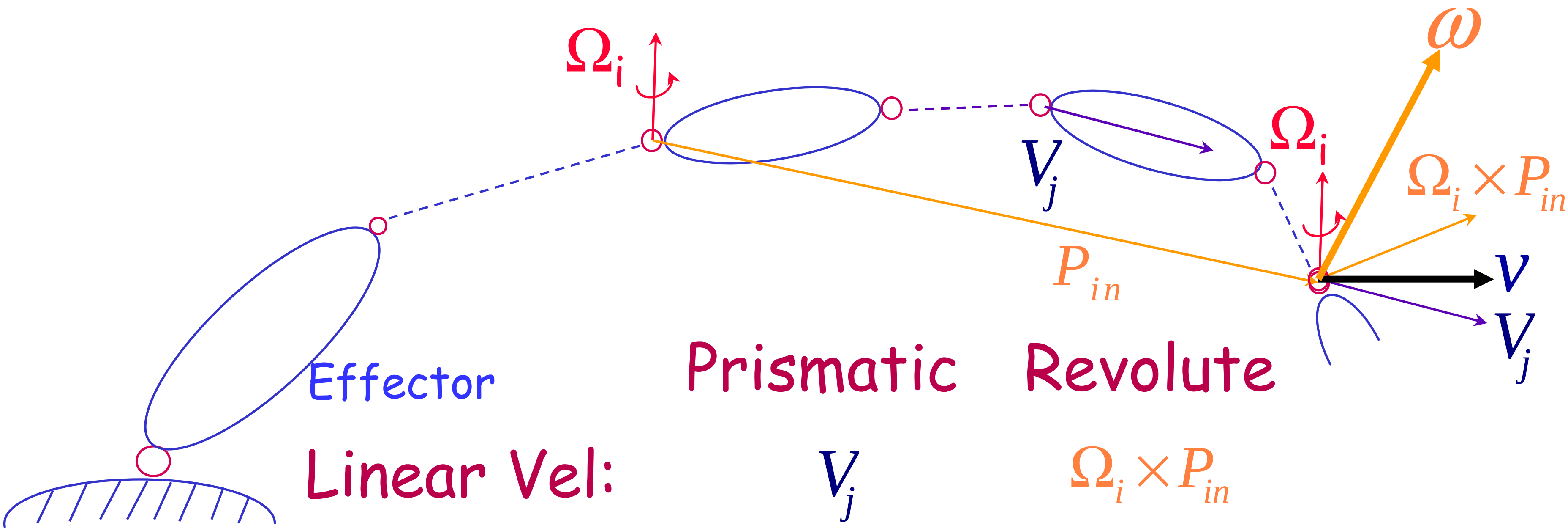

	Prismatic	Revolute
Linear Vel:	V_j	$\Omega_i \times P_{in}$
Angular Vel:	none	Ω_i

Effector Linear Velocity

$$v = \sum_{i=1}^{n} [\epsilon_i V_i + \bar{\epsilon}_i (\Omega_i \times P_{in})] \quad \Leftarrow \quad V_i = Z_i \dot{q}_i$$

Effector Angular Velocity

$$\omega = \sum_{i=1}^{n} \bar{\epsilon}_i \Omega_i \quad \Leftarrow \quad \Omega_i = Z_i \dot{q}_i$$

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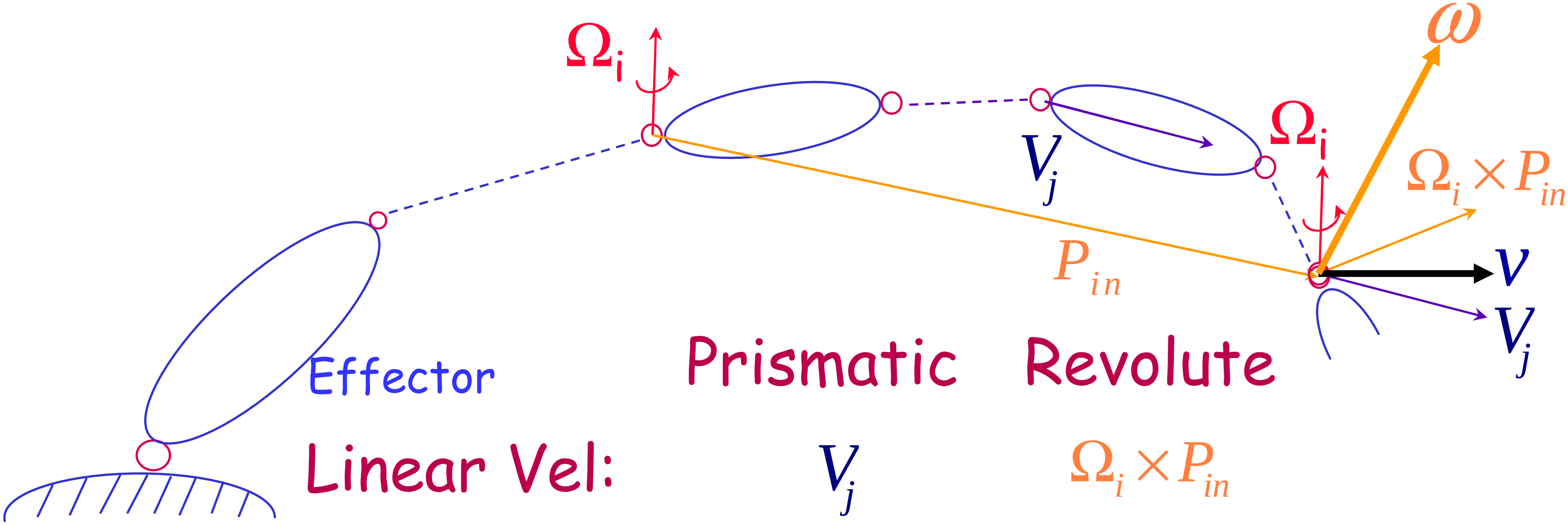


	Prismatic	Revolute
Linear Vel:	V_j	$\Omega_i \times P_{in}$
Angular Vel:	none	Ω_i

Effector Linear Velocity

$$v = \sum_{i=1}^{n} [\epsilon_i \, Z_i + \bar{\epsilon}_i \, (Z_i \times P_{in})] \dot{q}_i \quad \Leftarrow \quad V_i = Z_i \dot{q}_i$$

Effector Angular Velocity

$$\omega = \sum_{i=1}^{n} (\bar{\epsilon}_i \, Z_i) \, \dot{q}_i \quad \Leftarrow \quad \Omega_i = Z_i \dot{q}_i$$

$$v = [\in_1 Z_1 + \bar{\in}_1 (Z_1 \times P_{1n})]\dot{q}_1 + \cdots$$

$$+ [\in_{n-1} Z_{n-1} + \bar{\in}_{n-1} (Z_{n-1} \times P_{(n-1)n})]\dot{q}_{n-1} + \in_n Z_n \dot{q}_n$$

$$v = \begin{bmatrix} \in_1 Z_1 + \bar{\in}_1 (Z_1 \times P_{1n}) & \in_2 Z_2 + \bar{\in}_2 (Z_2 \times P_{2n}) & \cdots \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\boxed{v = J_v \dot{q}}$$

$$\omega = \bar{\in}_1 Z_1 \dot{q}_1 + \bar{\in}_2 Z_2 \dot{q}_2 + \cdots + \bar{\in}_n Z_n \dot{q}_n$$

$$\omega = \begin{bmatrix} \bar{\in}_1 Z_1 & \bar{\in}_2 Z_2 & \cdots & \bar{\in}_n Z_n \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\boxed{\omega = J_\omega \dot{q}}$$

The Jacobian

$$J = \begin{pmatrix} J_v \\ \hline J_w \end{pmatrix}$$

Matrix J_v (direct differentiation)

$$v = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \dot{x}_P = \frac{\partial x_P}{\partial q_1}.\dot{q}_1 + \frac{\partial x_P}{\partial q_2}.\dot{q}_2 + \cdots + \frac{\partial x_P}{\partial q_n}.\dot{q}_n$$

$$J_v = \begin{pmatrix} \frac{\partial x_P}{\partial q_1} & \frac{\partial x_P}{\partial q_2} & \cdots & \frac{\partial x_P}{\partial q_n} \end{pmatrix}$$

Jacobian in a Frame

Vector Representation

$$J = \begin{pmatrix} \dfrac{\partial x_P}{\partial q_1} & \dfrac{\partial x_P}{\partial q_2} & \cdots & \dfrac{\partial x_P}{\partial q_n} \\ \bar{\epsilon}_1 . Z_1 & \bar{\epsilon}_2 . Z_2 & \cdots & \bar{\epsilon}_n . Z_n \end{pmatrix}$$

In {0}

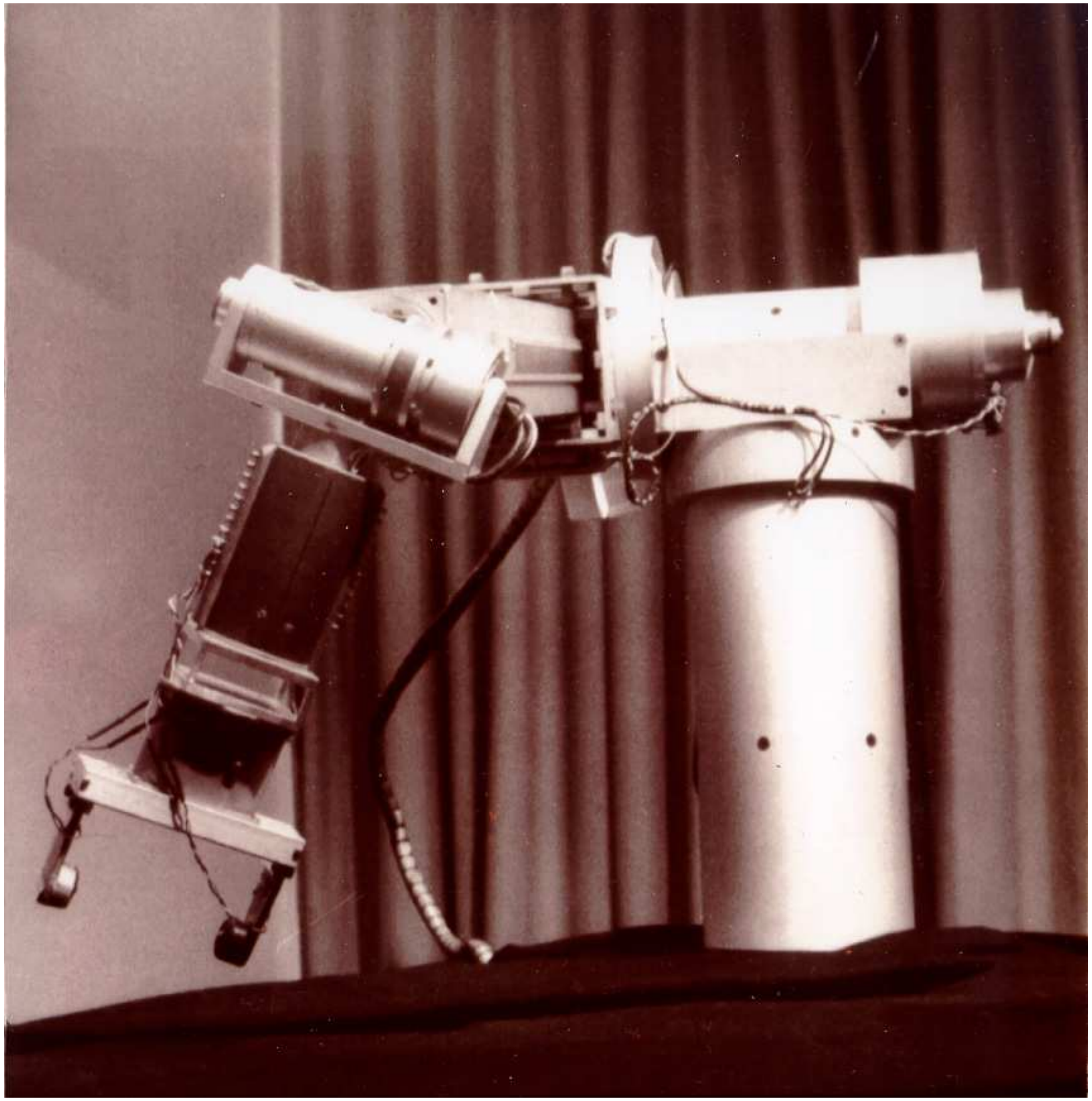
$${}^0 J = \begin{pmatrix} \dfrac{\partial {}^0 x_P}{\partial q_1} & \dfrac{\partial {}^0 x_P}{\partial q_2} & \cdots & \dfrac{\partial {}^0 x_P}{\partial q_n} \\ \bar{\epsilon}_1 . {}^0 Z_1 & \bar{\epsilon}_2 . {}^0 Z_2 & \cdots & \bar{\epsilon}_n . {}^0 Z_n \end{pmatrix}$$

J in Frame {0}

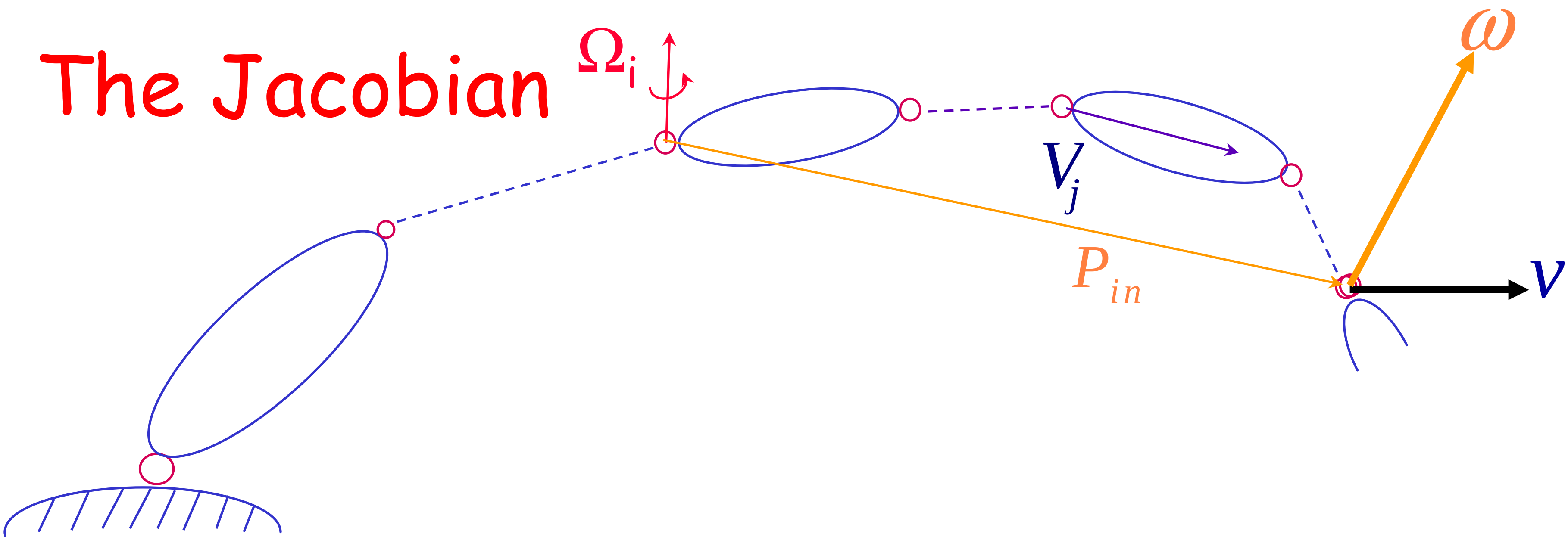
$$ {}^{0}Z_i = {}^{0}_{i}R \; {}^{i}Z_i ; \quad {}^{i}Z_i = Z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} $$

$$ {}^{0}J = \begin{pmatrix} \frac{\partial}{\partial q_1}({}^{0}x_P) & \frac{\partial}{\partial q_2}({}^{0}x_P) & \cdots & \frac{\partial}{\partial q_n}({}^{0}x_P) \\ \overline{\in}_1 . ({}^{0}_{1}R . Z) & \overline{\in}_2 . ({}^{0}_{2}R . Z) & \cdots & \overline{\in}_n . ({}^{0}_{n}R . Z) \end{pmatrix} $$

Stanford Scheinman Arm



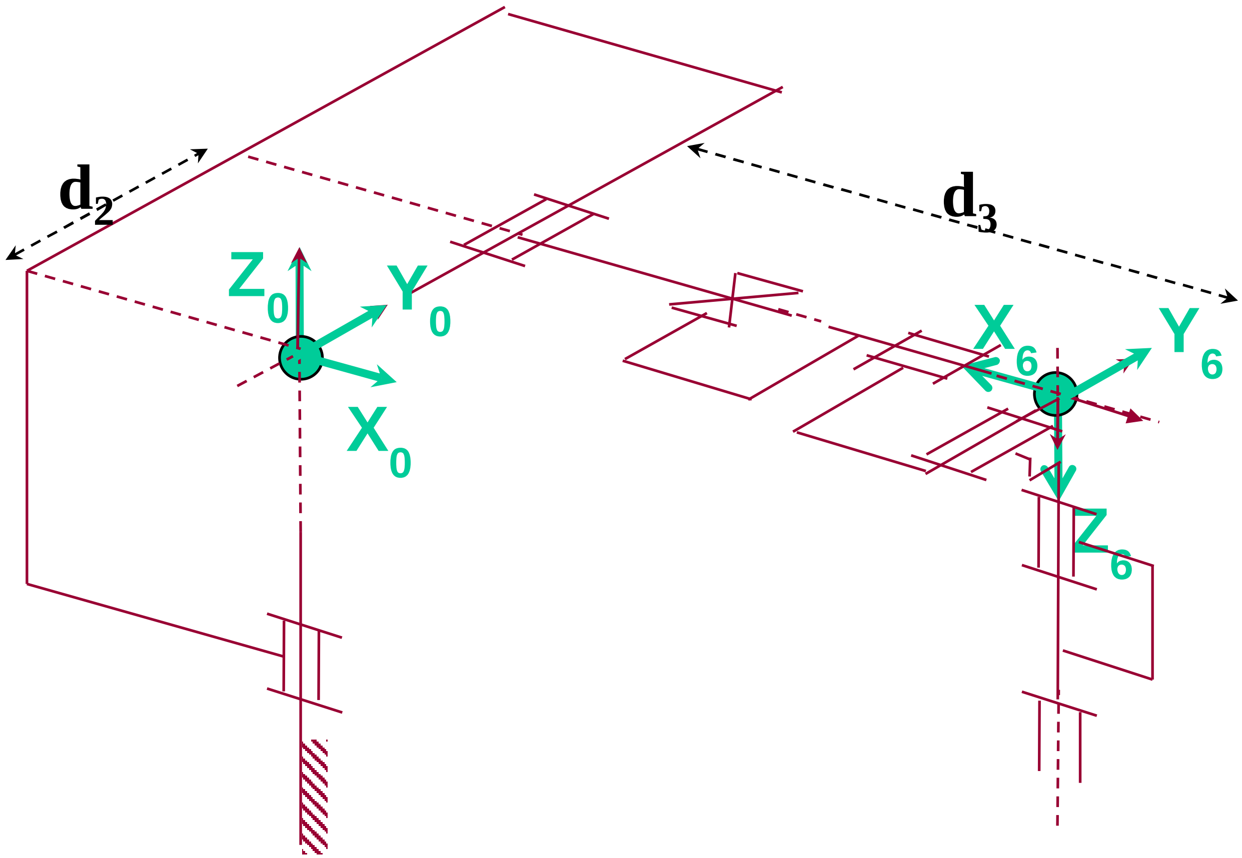
The Jacobian



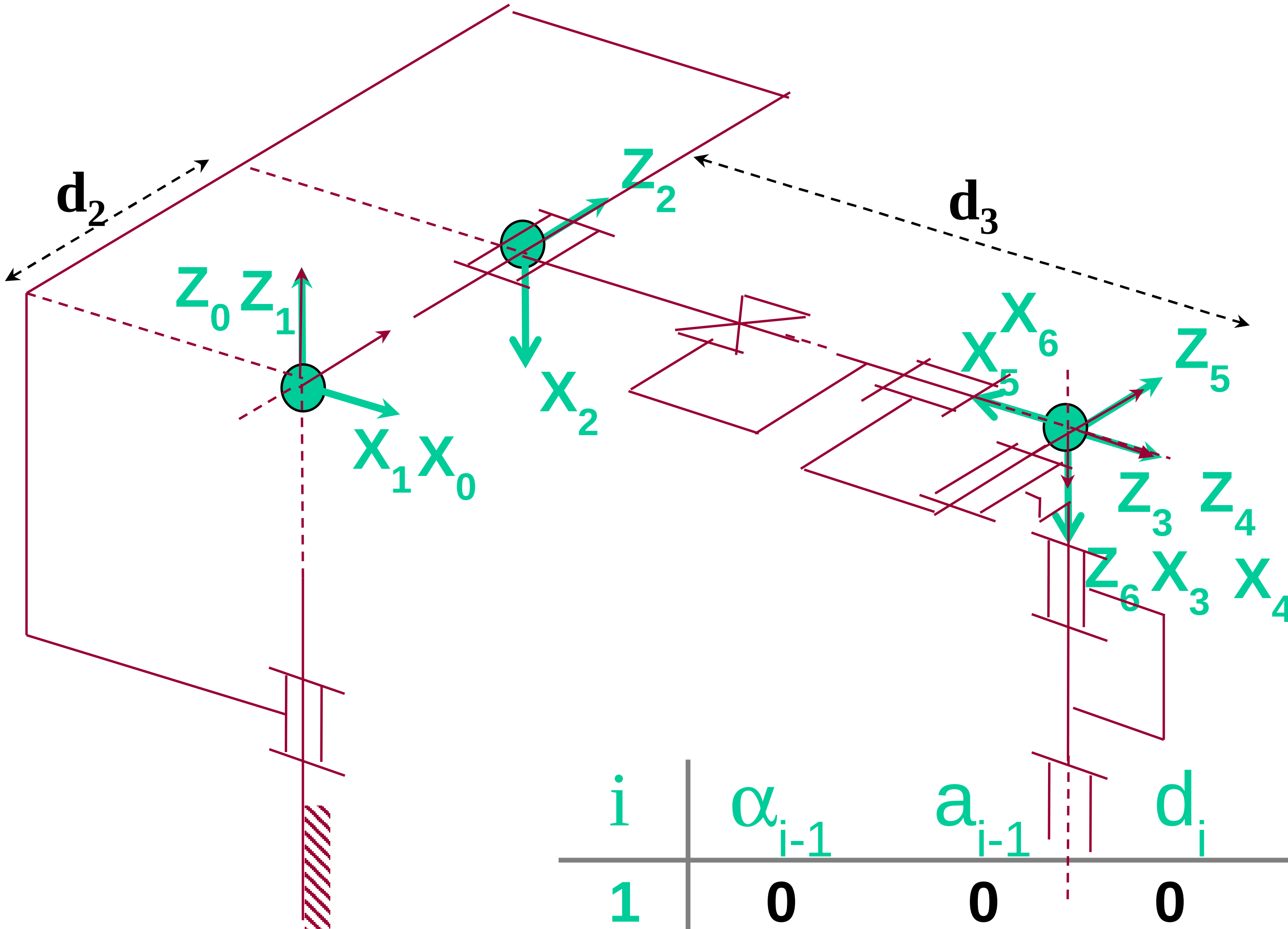
$$J = \begin{pmatrix} J_v \\ \hline J_w \end{pmatrix} \qquad v = J_v \dot{q} \qquad \omega = J_\omega \dot{q}$$

$$J_v = \left[\in_1 Z_1 + \overline{\in}_1 (Z_1 \times P_{1n}) \quad \in_2 Z_2 + \overline{\in}_2 (Z_2 \times P_{2n}) \quad \cdots \right]$$

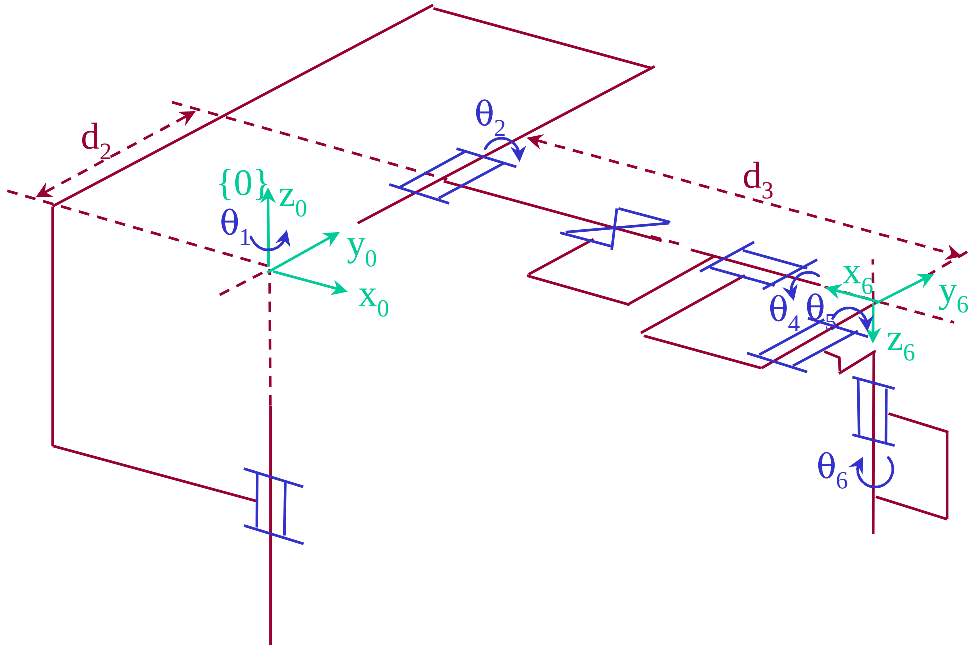
$$J_\omega = \left[\overline{\in}_1 Z_1 \quad \overline{\in}_2 Z_2 \quad \cdots \quad \overline{\in}_n Z_n \right]$$



$$J = \begin{pmatrix} Z_1 \times P_{13} & Z_2 \times P_{23} & Z_3 & 0 & 0 & 0 \\ Z_1 & Z_2 & 0 & Z_4 & Z_5 & Z_6 \end{pmatrix}$$



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	d_3	0
4	0	0	0	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

$$
{}^{i-1}_{\;\;\;i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics: $${}^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; \dots \; {}^{N-1}_{\;\;\;N}T$$

Stanford Scheinman Arm

$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}_{6}T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$ {}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{2}T = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & -s_1 & -s_1 d_2 \\ s_1 c_2 & -s_1 s_2 & c_1 & c_1 d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

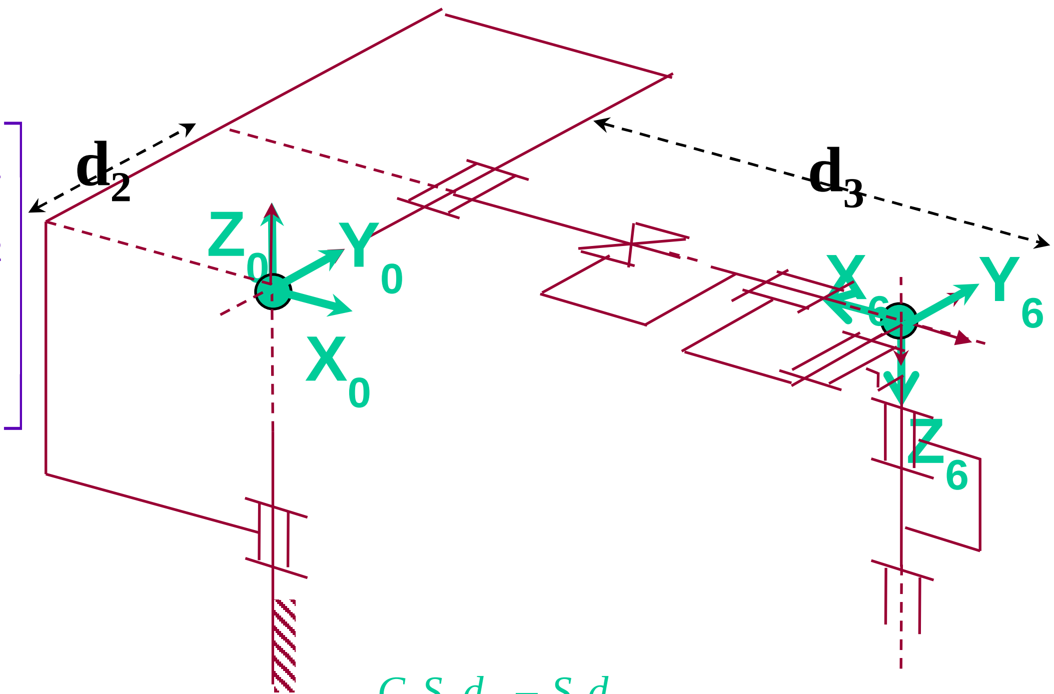
$$ {}^{0}_{3}T = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & c_1 d_3 s_2 - s_1 d_2 \\ s_1 c_2 & c_1 & s_1 s_2 & s_1 d_3 s_2 + c_1 d_2 \\ -s_2 & 0 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{0}_{4}T = \begin{bmatrix} c_1c_2c_4 - s_1s_4 & -c_1c_2s_4 - s_1c_4 & c_1s_2 & c_1d_3s_2 - s_1d_2 \\ s_1c_2c_4 + c_1s_4 & -s_1c_2s_4 + c_1c_4 & s_1s_2 & s_1d_3s_2 + c_1d_2 \\ -s_2c_4 & s_2s_4 & c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{5}T = \begin{bmatrix} X & X & -c_1c_2s_4 - s_1c_4 & c_1d_3s_2 - s_1d_2 \\ X & X & -s_1c_2s_4 + c_1c_4 & s_1d_3s_2 + c_1d_2 \\ X & X & s_2s_4 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{6}T = \begin{bmatrix} X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\ X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\ X & X & -s_2c_4s_5 + c_5c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{6}T = \begin{bmatrix} X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\ X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\ X & X & -s_2c_4s_5 + c_5c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



$$
x = \begin{pmatrix} x_P \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = \left| \begin{array}{c}
C_1S_2d_3 - S_1d_2 \\
S_1S_2d_3 + C_1d_2 \\
C_2d_3 \\
C_1[C_2(C_4C_5C_6 - S_4S_6) - S_2S_5C_6] - S_1(S_4C_5C_6 + C_4S_6) \\
S_1[C_2(C_4C_5C_6 - S_4S_6) - S_2S_5C_6] + C_1(S_4C_5C_6 + C_4S_6) \\
-S_2(C_4C_5C_6 - S_4S_6) - C_2S_5C_6 \\
C_1[-C_2(C_4C_5S_6 + S_4C_6) + S_2S_5S_6] - S_1(-S_4C_5S_6 + C_4C_6) \\
S_1[-C_2(C_4C_5S_6 + S_4C_6) + S_2S_5S_6] + C_1(-S_4C_5S_6 + C_4C_6) \\
S_2(C_4C_5S_6 + S_4C_6) + C_2S_5S_6 \\
C_1(C_2C_4S_5 + S_2C_5) - S_1S_4S_5 \\
S_1(C_2C_4S_5 + S_2C_5) + C_1S_4S_5 \\
-S_2C_4S_5 + C_2C_5
\end{array} \right|
$$

Stanford Scheinman Arm Jacobian

$$ {}^{0}J = \begin{pmatrix} \dfrac{\partial {}^{0}x_P}{\partial q_1} & \dfrac{\partial {}^{0}x_P}{\partial q_2} & \dfrac{\partial {}^{0}x_P}{\partial q_3} & 0 & 0 & 0 \\ {}^{0}Z_1 & {}^{0}Z_2 & 0 & {}^{0}Z_4 & {}^{0}Z_5 & {}^{0}Z_6 \end{pmatrix} $$

$$ \begin{bmatrix} -c_1 d_2 - s_1 s_2 d_3 & c_1 c_2 d_3 & c_1 s_2 & 0 & 0 & 0 \\ -s_1 d_2 + c_1 s_2 d_3 & s_1 c_2 d_3 & s_1 s_2 & 0 & 0 & 0 \\ 0 & -s_2 d_3 & c_2 & 0 & 0 & 0 \\ 0 & -s_1 & 0 & c_1 s_2 & -c_1 c_2 s_4 - s_1 c_4 & c_1 c_2 c_4 s_5 - s_1 s_4 s_5 + c_1 s_2 c_5 \\ 0 & c_1 & 0 & s_1 s_2 & -s_1 c_2 s_4 + c_1 c_4 & s_1 c_2 c_4 s_5 + c_1 s_4 s_5 + s_1 s_2 c_5 \\ 1 & 0 & 0 & c_2 & s_2 s_4 & -s_2 c_4 s_5 + c_5 c_2 \end{bmatrix} $$