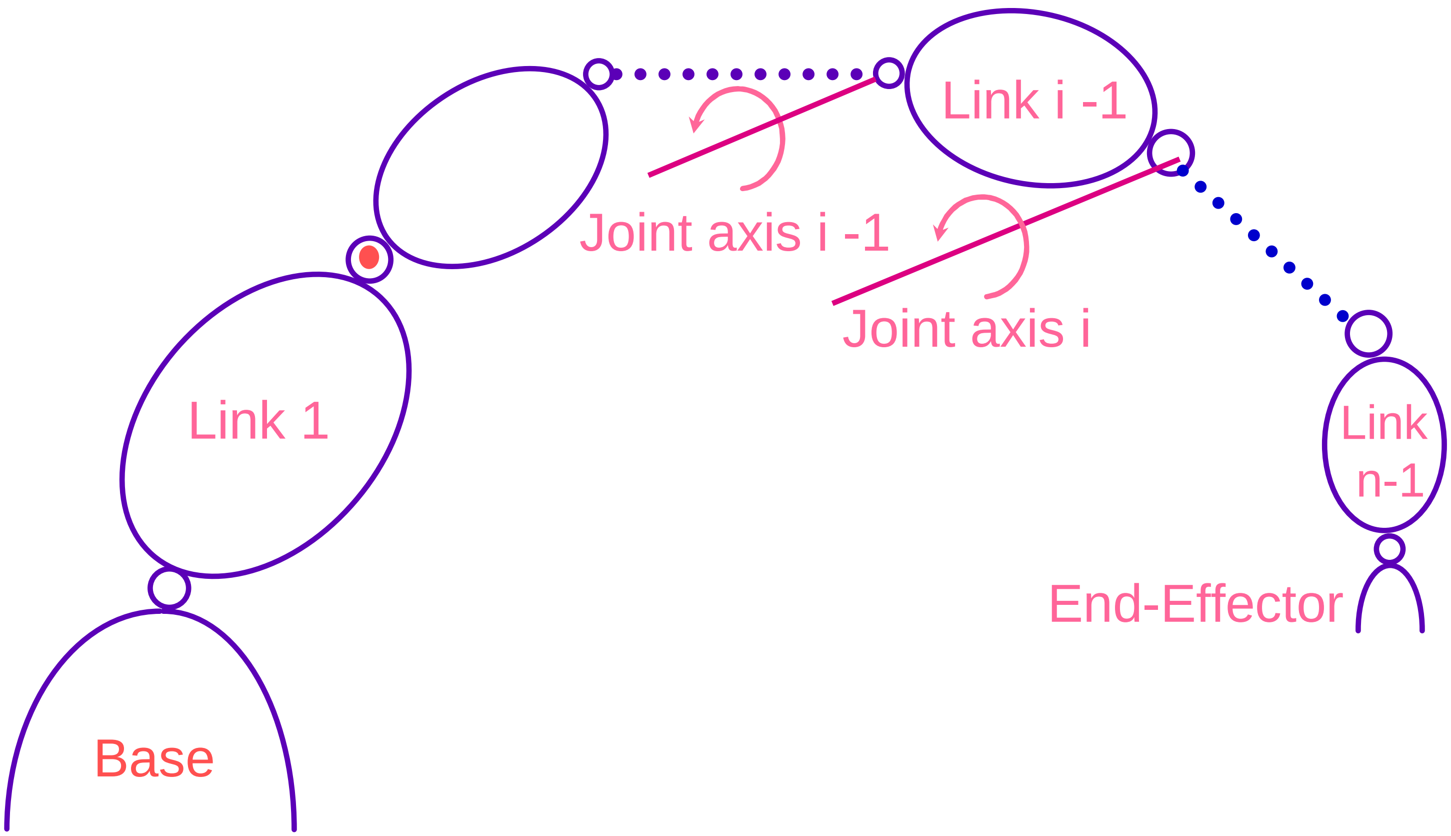
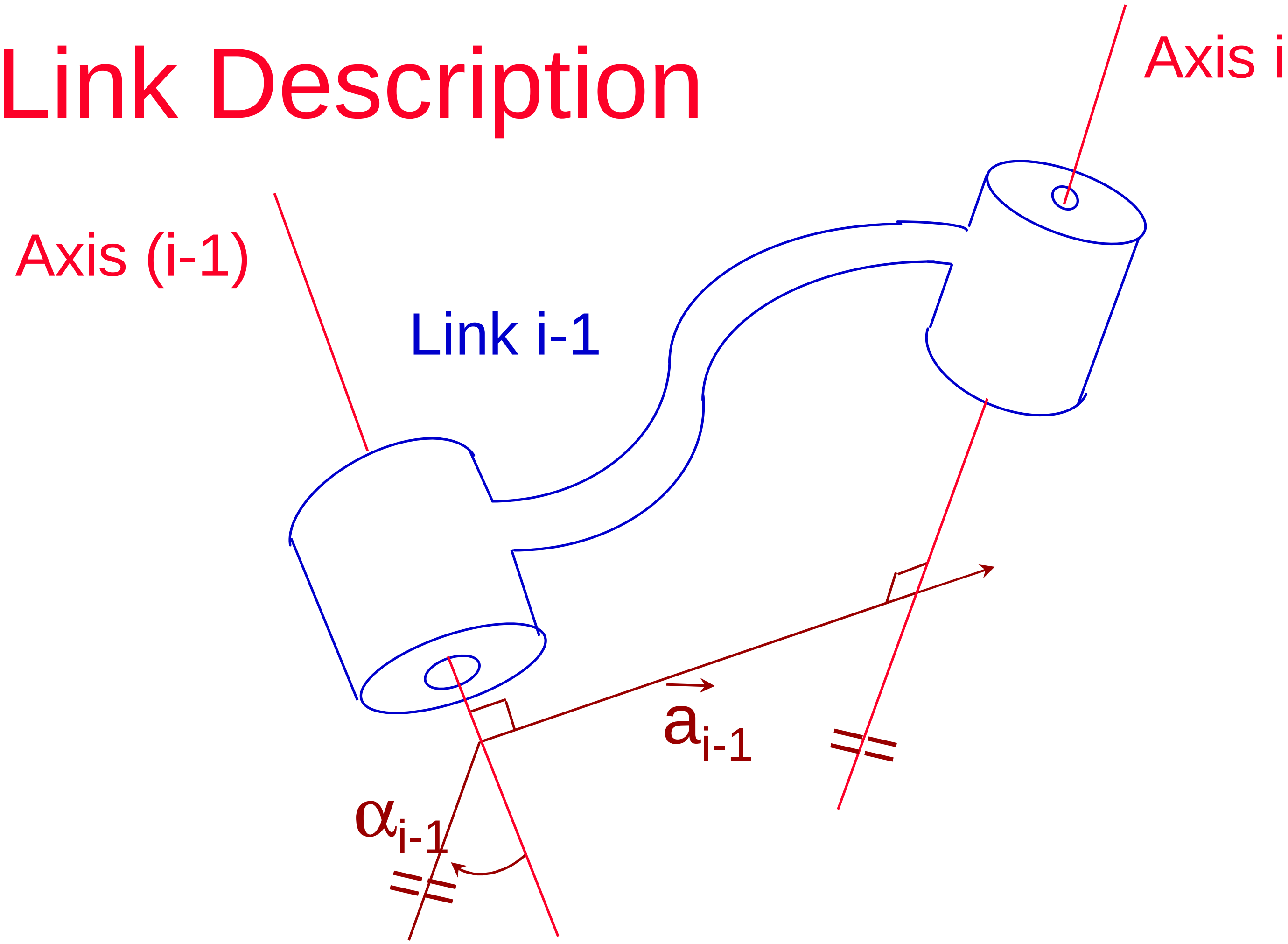
Manipulator Kinematics

- Link Description
- *Denavit-Hartenberg* Notation
- Frame Attachment
- Forward Kinematics

Manipulator



Link Description

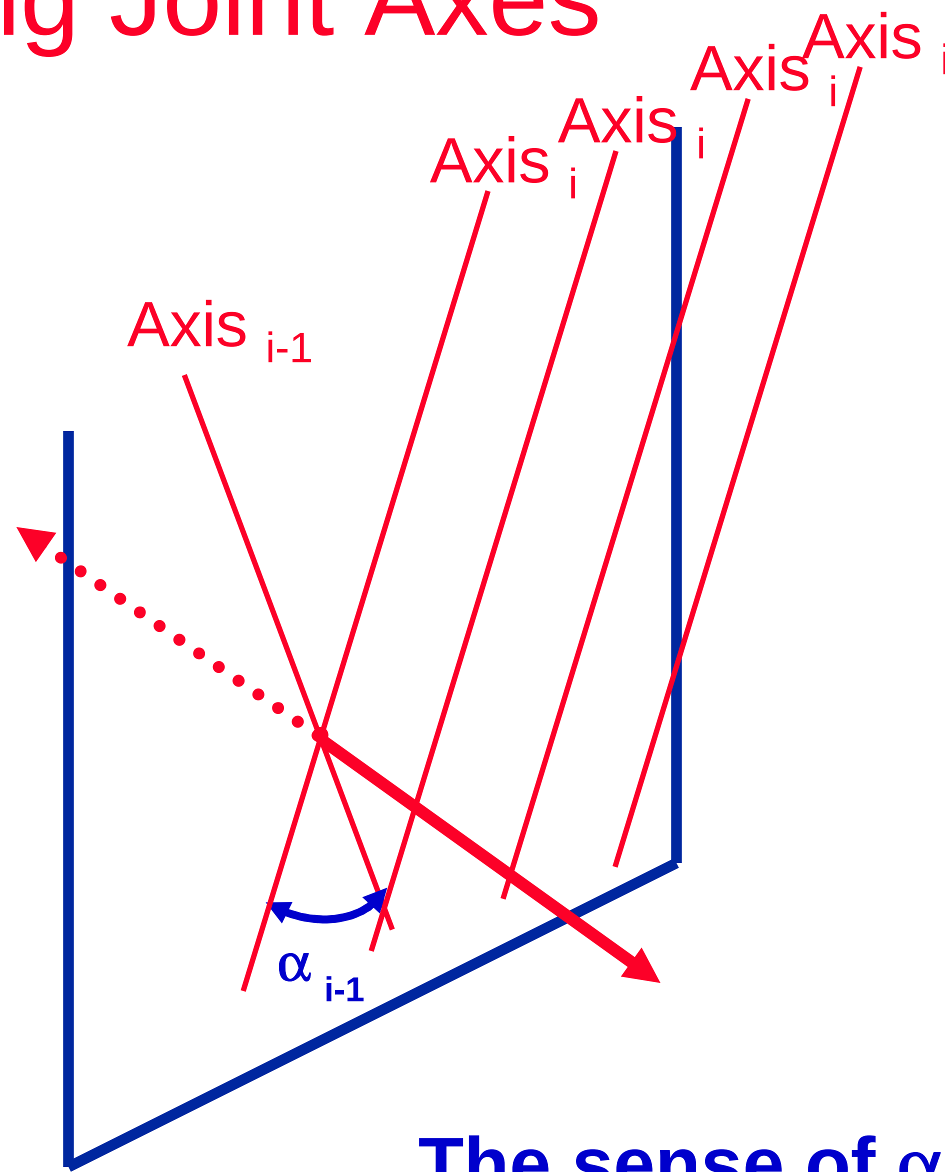


a_{i-1}: Link Length - mutual perpendicular

unique except for parallel axis

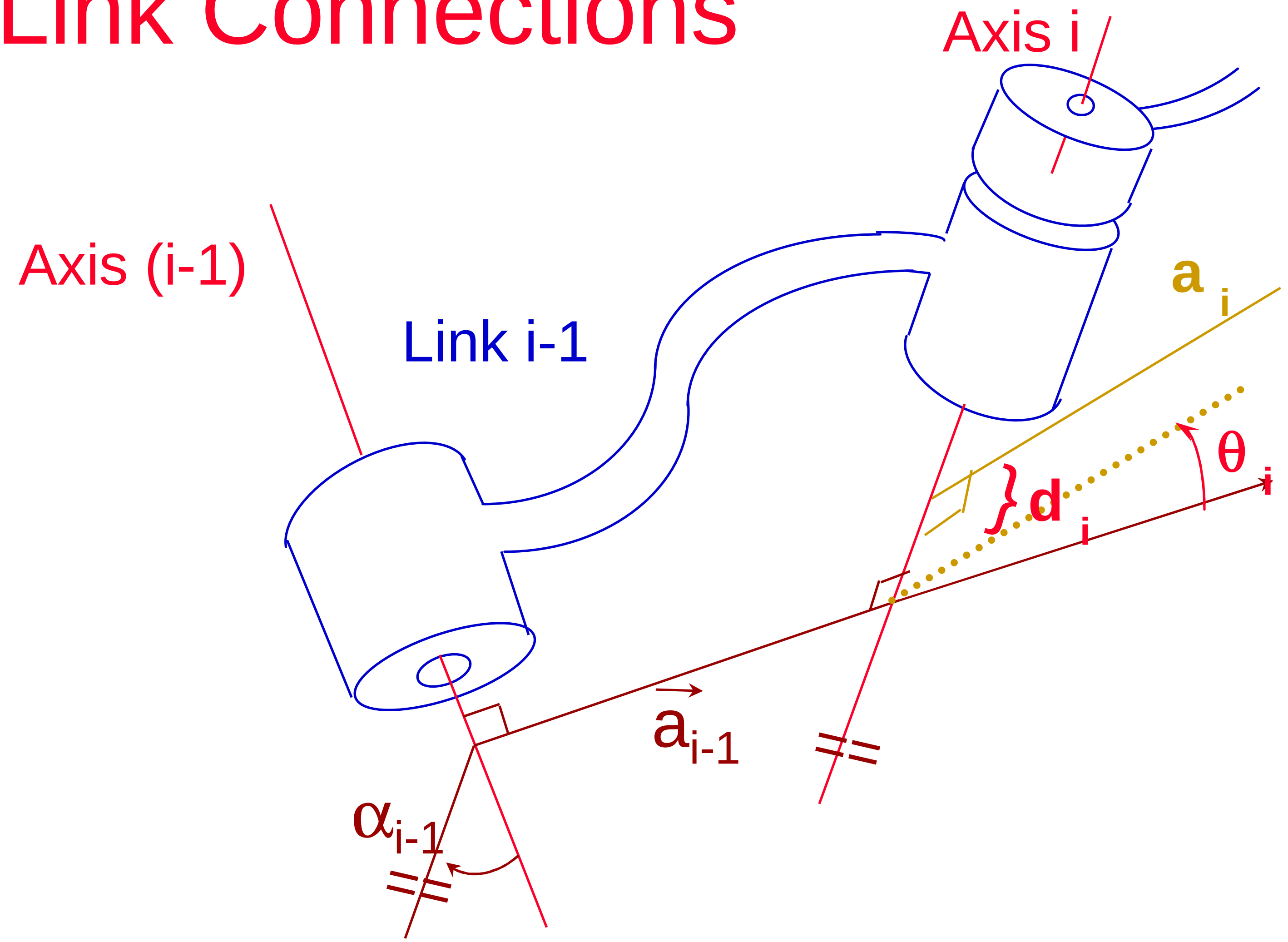
α_{i-1}: Link Twist - measured in the right-hand sense about $\vec{a}_{i-1}$

Intersecting Joint Axes



The sense of α_{i-1} is free

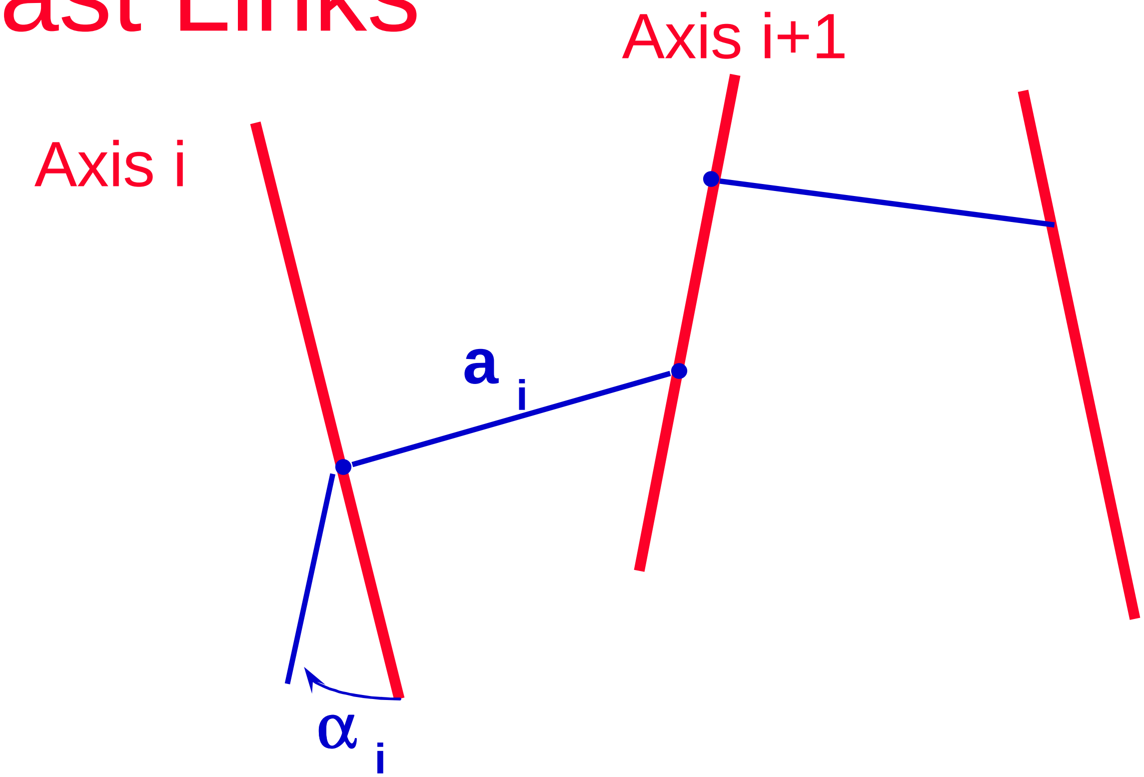
Link Connections



d_i: Link Offset -- variable if joint i is *prismatic*

θ_i: Joint Angle -- variable if joint i is *revolute*

First & Last Links



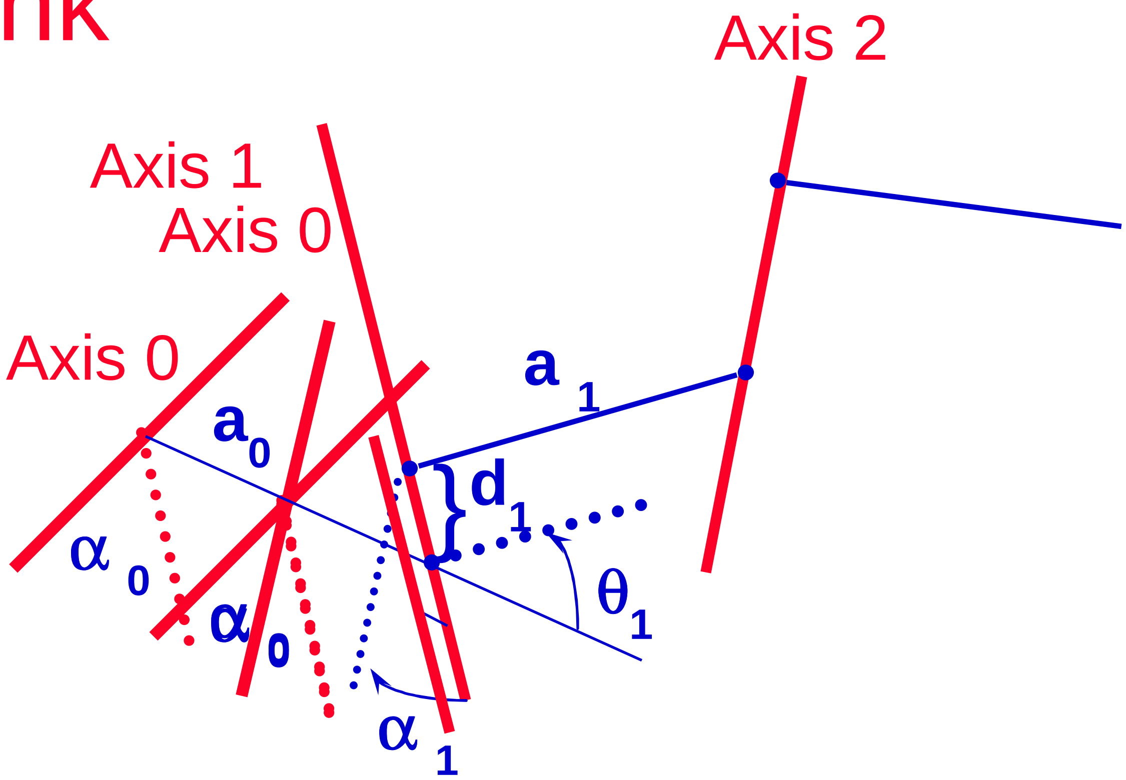
a_i and α_i depend on joint axes i and i+1

Axes 1 to n: determined

⇒ a_1 , a_2 ... a_{n-1} and α_1, α_2 ... α_{n-1}

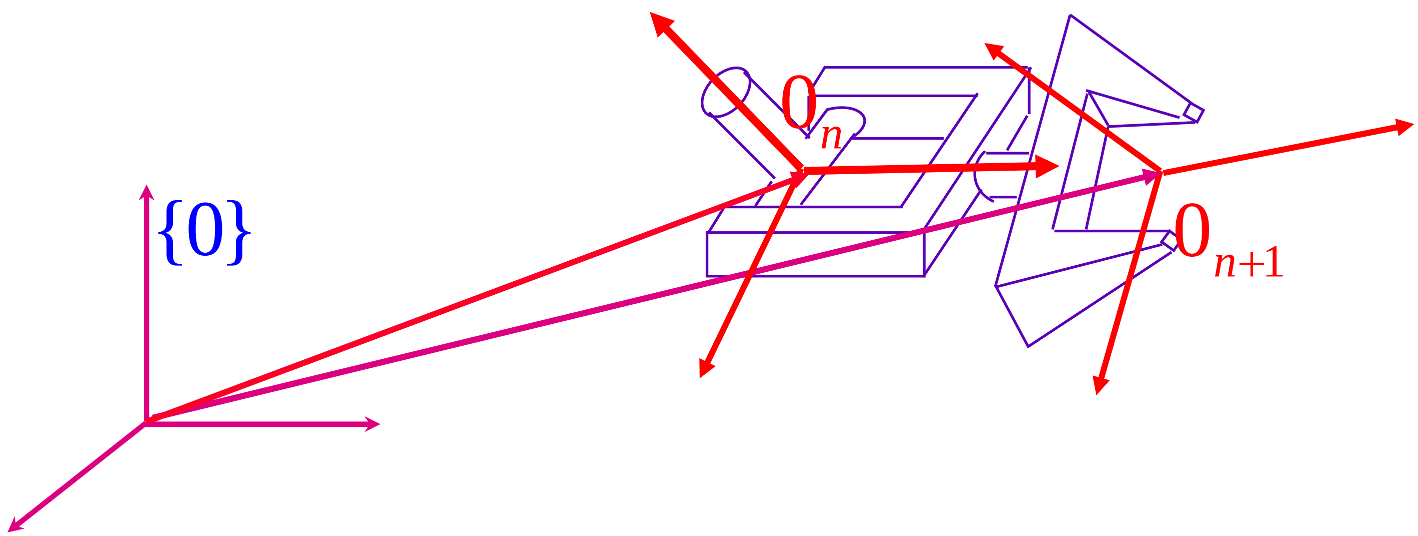
Convention: $a_0 = a_n = 0$ and $\alpha_0 = \alpha_n = 0$

First Link

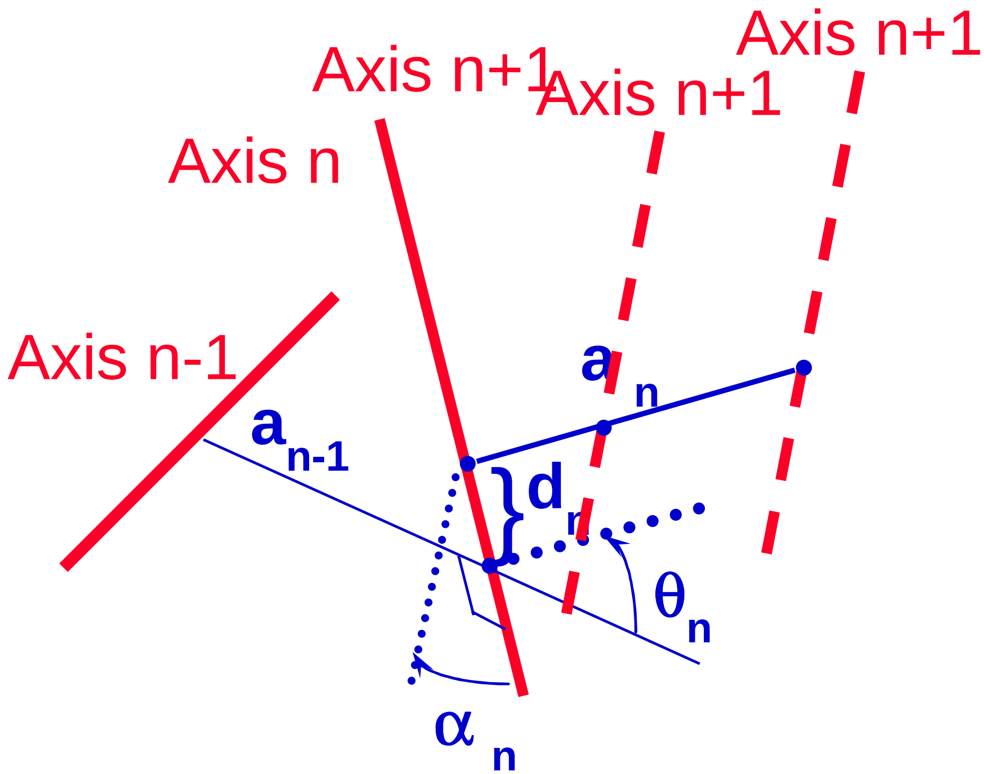


$$a_0 = 0 \text{ and } \alpha_0 = 0$$

End-Efector Frame

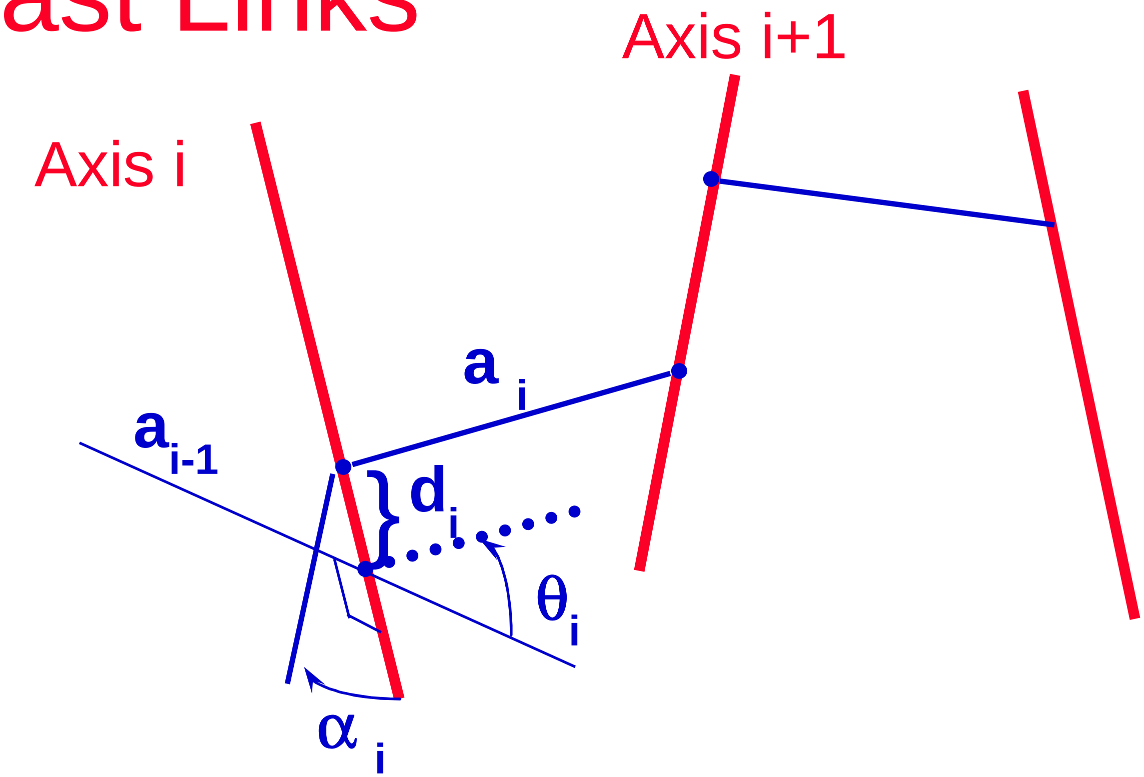


Last Link



$$a_n = 0 \quad \text{and} \quad \alpha_n = 0$$

First & Last Links



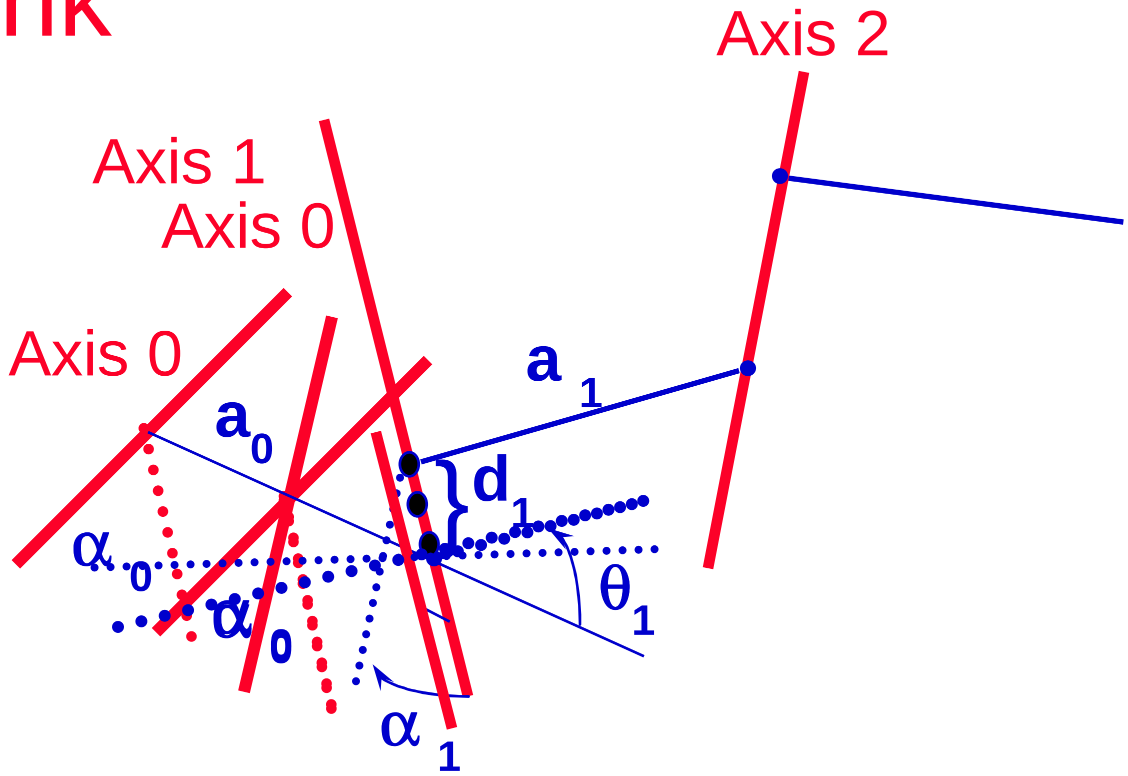
θ_i and d_i depend on links i-1 and i

⇒ $\theta_2, \theta_3, \ldots, \theta_{n-1}$ and $d_2, d_3, \ldots, d_{n-1}$

Convention: set the constant parameters to zero

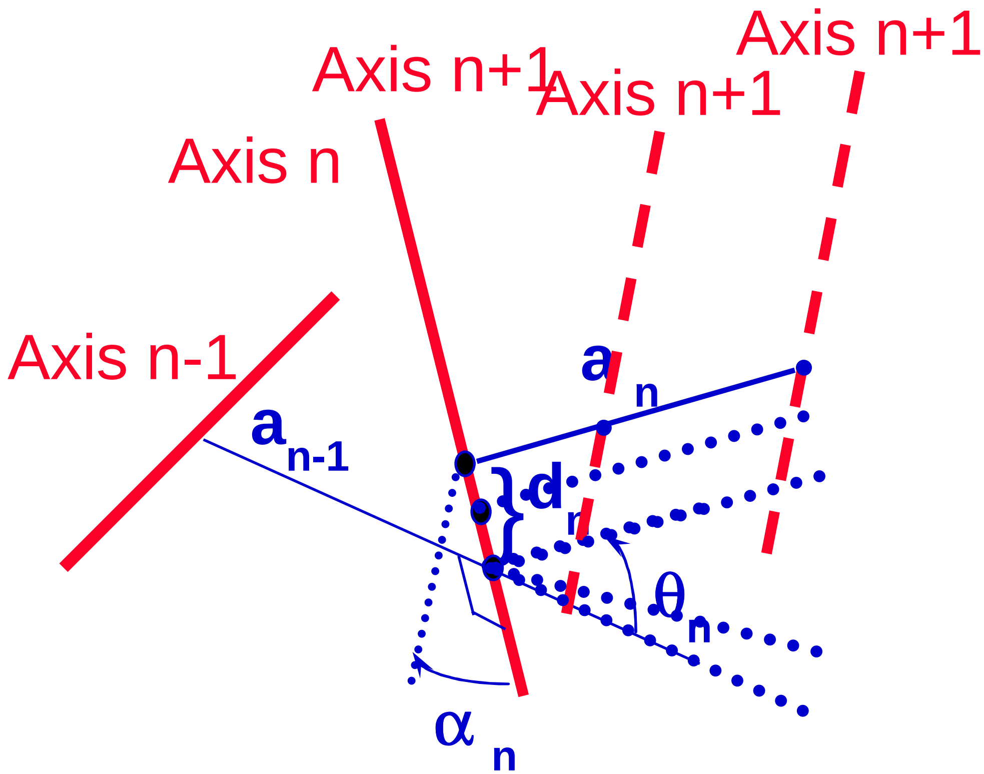
Following joint type: d_1 or $\theta_1 = 0$ and d_n or $\theta_n = 0$

First Link



d_1 or $\theta_1 = 0$

Last Link



d_n or $\theta_n = 0$

Denavit-Hartenberg Parameters

4 D-H parameters $(\alpha_i\ ,\ \ a_i\ ,\ \ d_i\ ,\ \ \theta_i)$

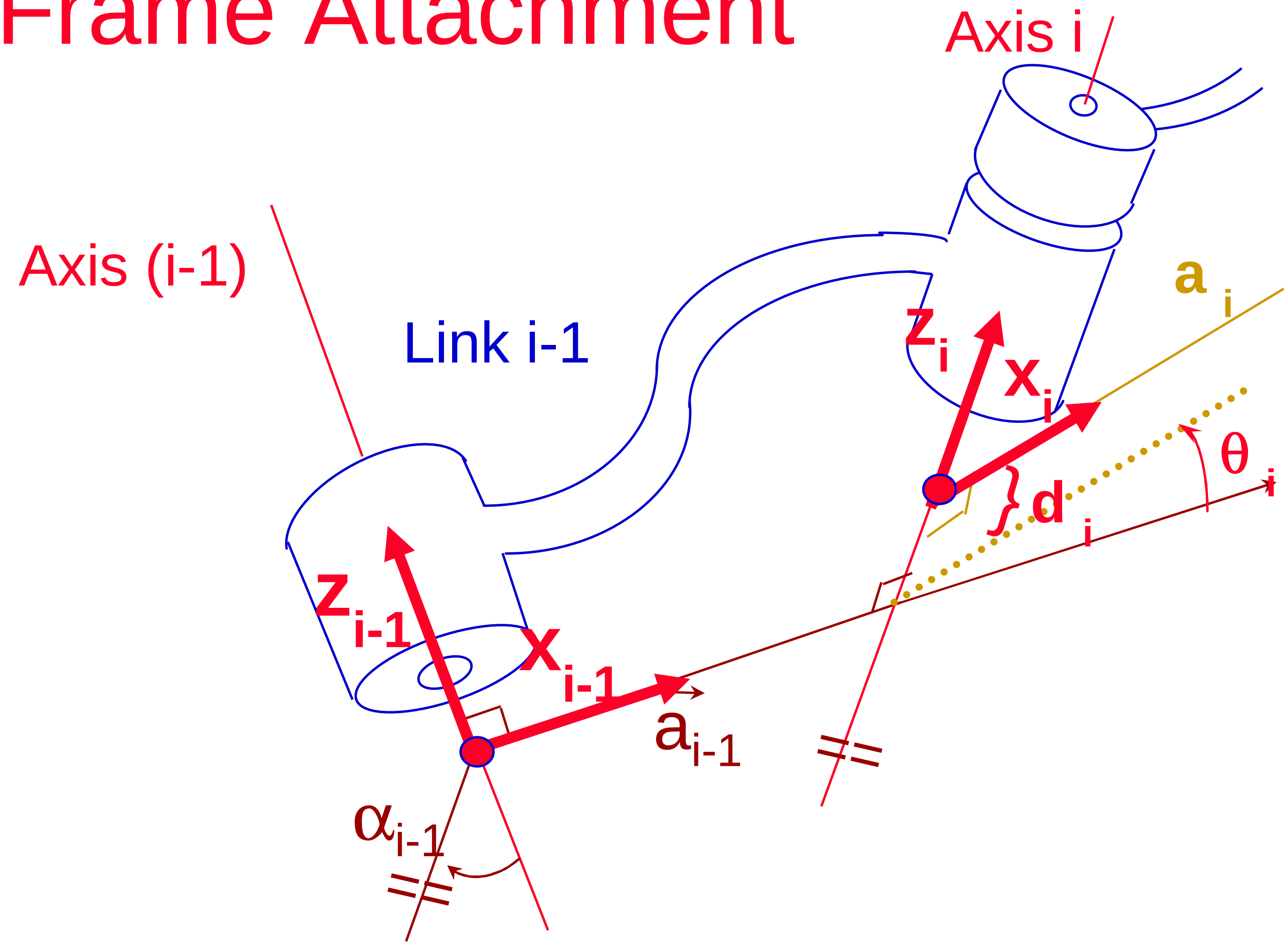
3 fixed link parameters

1 joint variable $\begin{cases} \theta_i \text{ revolute joint} \\ d_i \text{ prismatic joint} \end{cases}$

α_i and a_i : describe the Link i

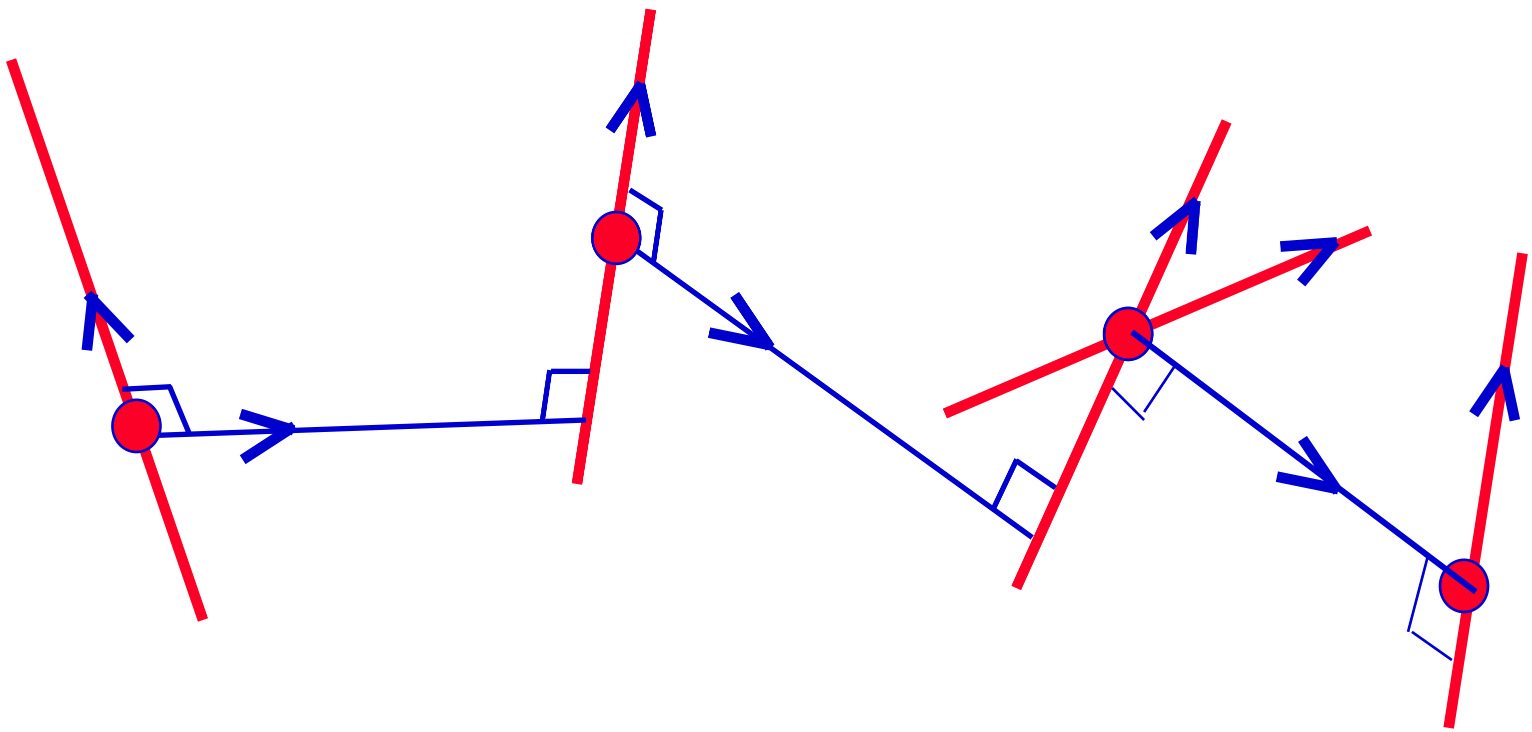
d_i and θ_i : describe the Link’s connection

Frame Attachment



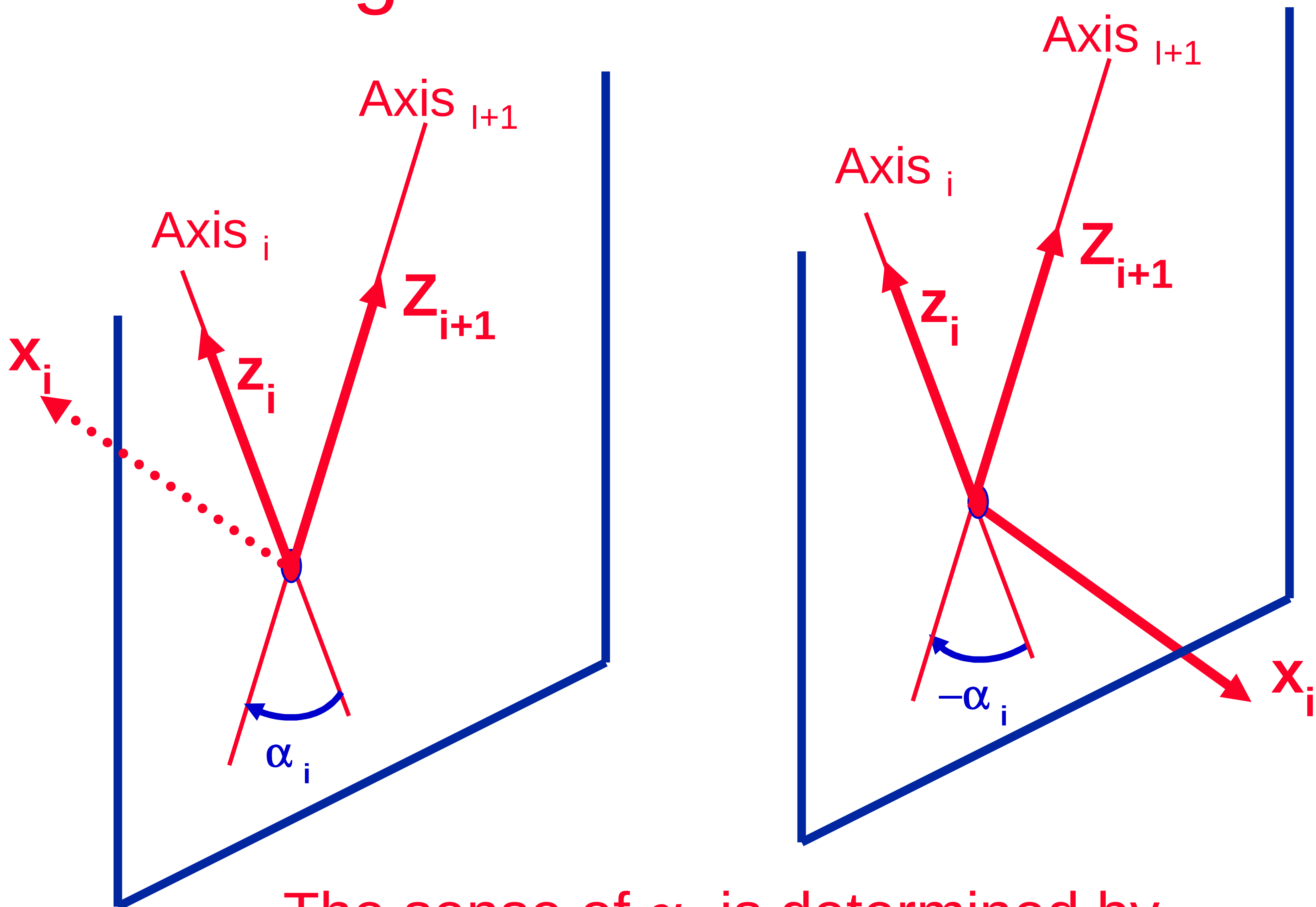
y-vectors: complete right-hand frames

Summary – Frame Attachment



1. Normals
2. Origins
3. Z-axes
4. X-axes

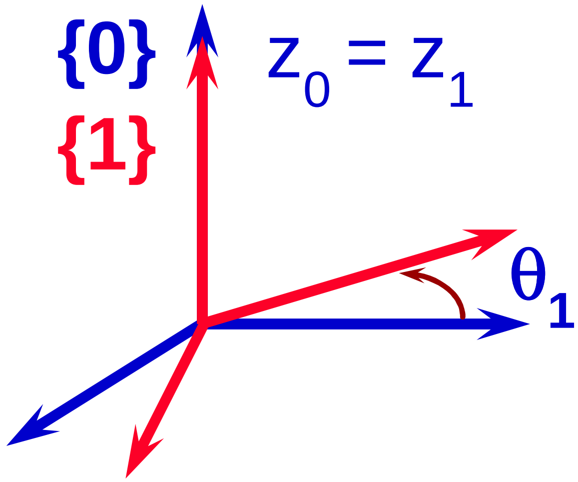
Intersecting Joint Axes



The sense of α_i is determined by the direction of x

First Link

Revolute



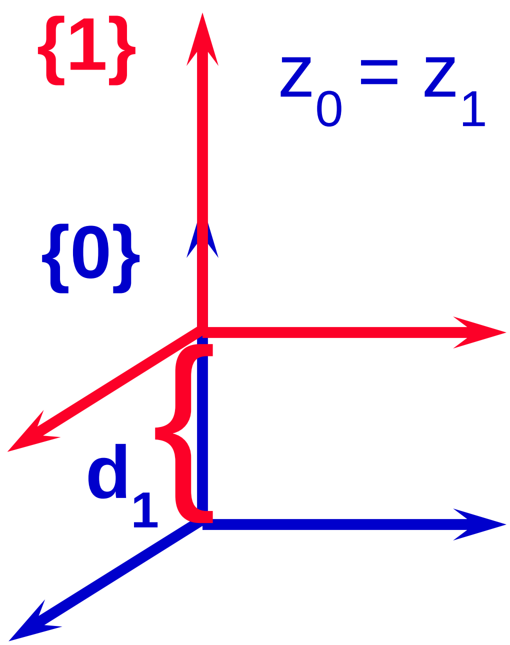
$a_0 = 0$

$\alpha_0 = 0$

$d_1 = 0$

$\theta_1 = 0 \longrightarrow \{0\} \equiv \{1\}$

Prismatic



$a_0 = 0$

$\alpha_0 = 0$

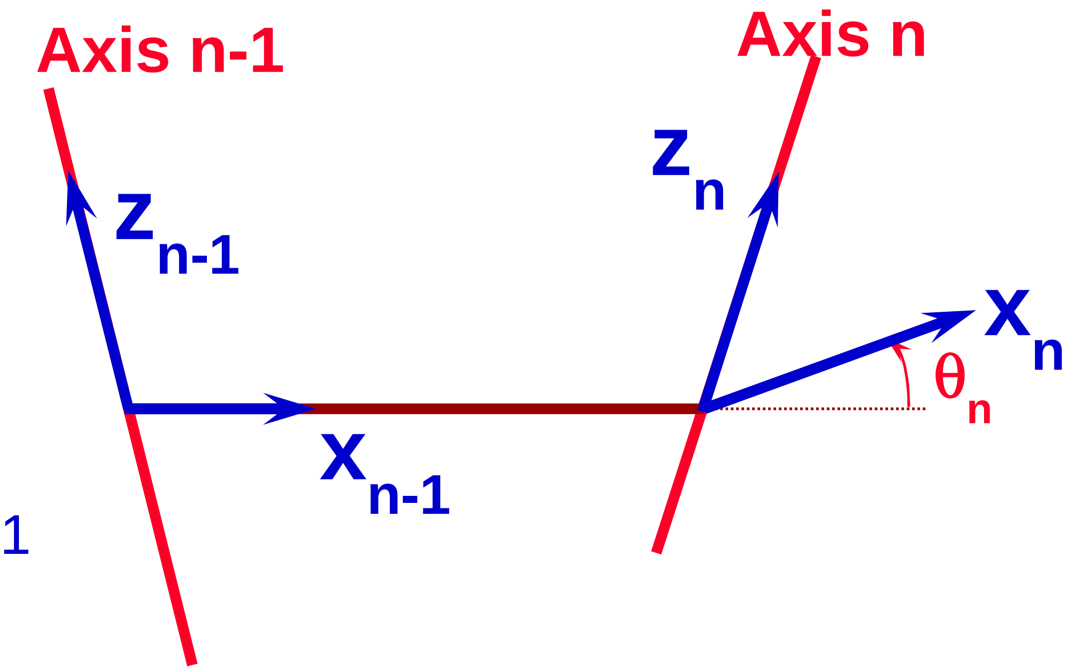
$\theta_1 = 0$

$\mathbf{d_1 = 0} \longrightarrow \mathbf{\{0\} \equiv \{1\}}$

Last Link

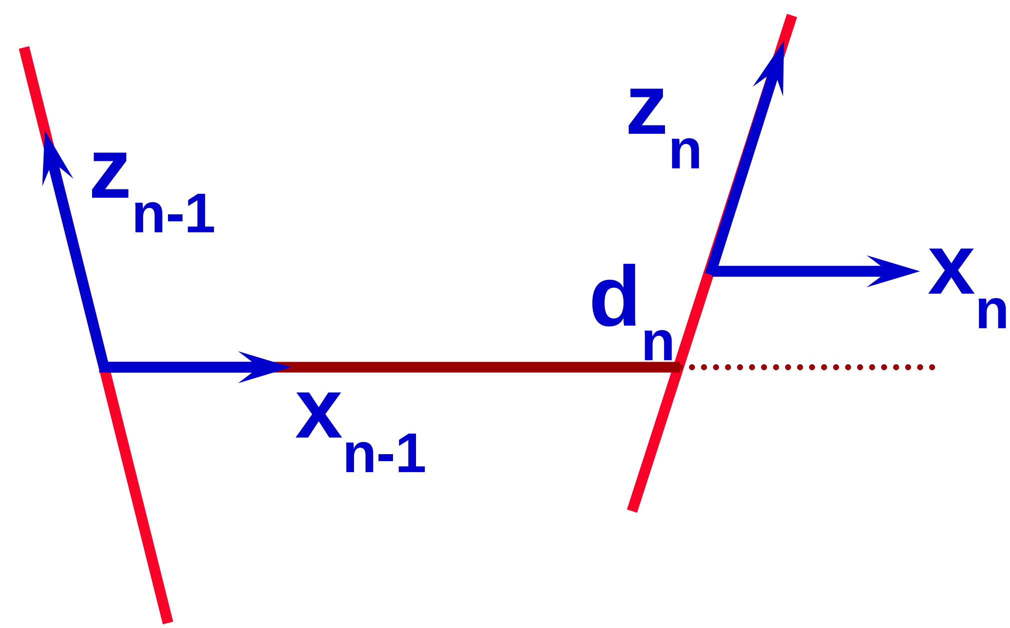
Revolute $d_n = 0$

$\theta_n = 0 \rightarrow x_n = x_{n-1}$

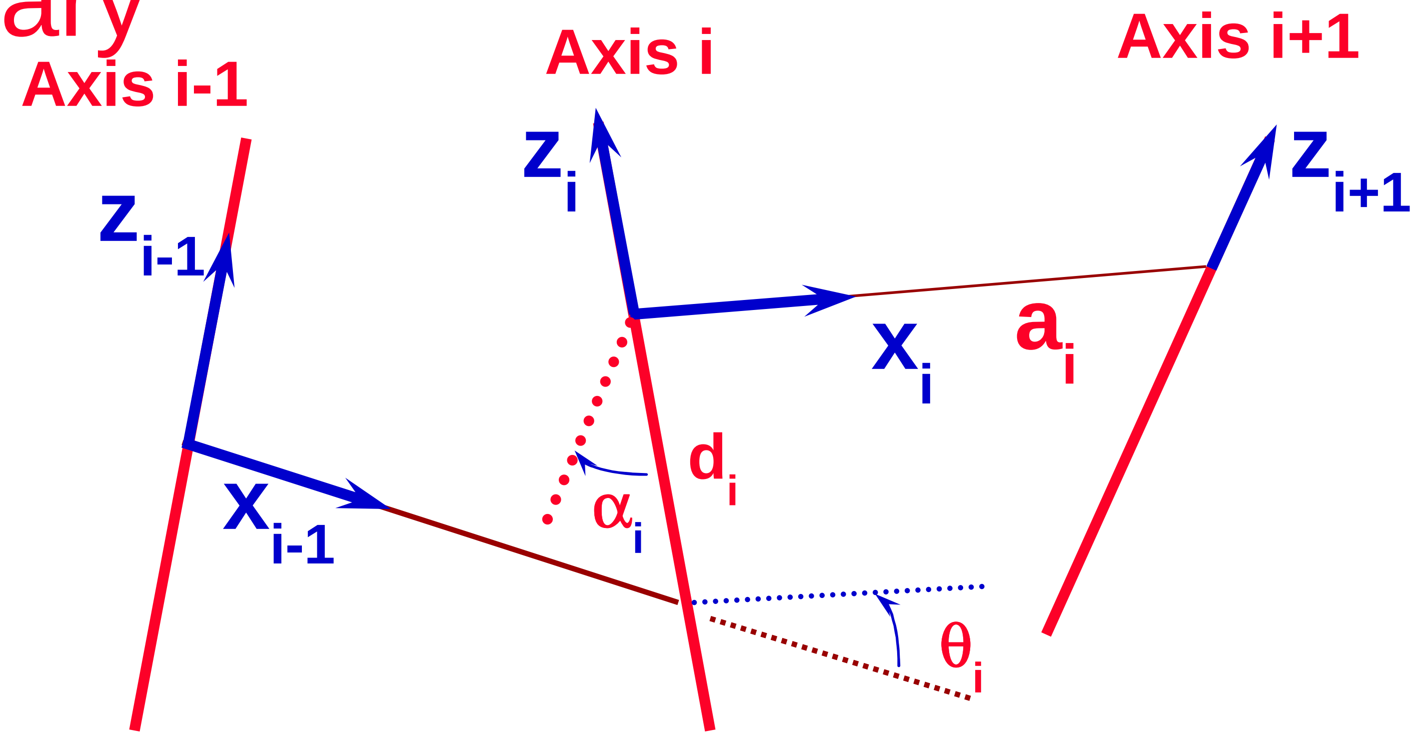


Prismatic $\theta_n = 0$

$d_n = 0 \rightarrow x_n = x_{n-1}$



Summary



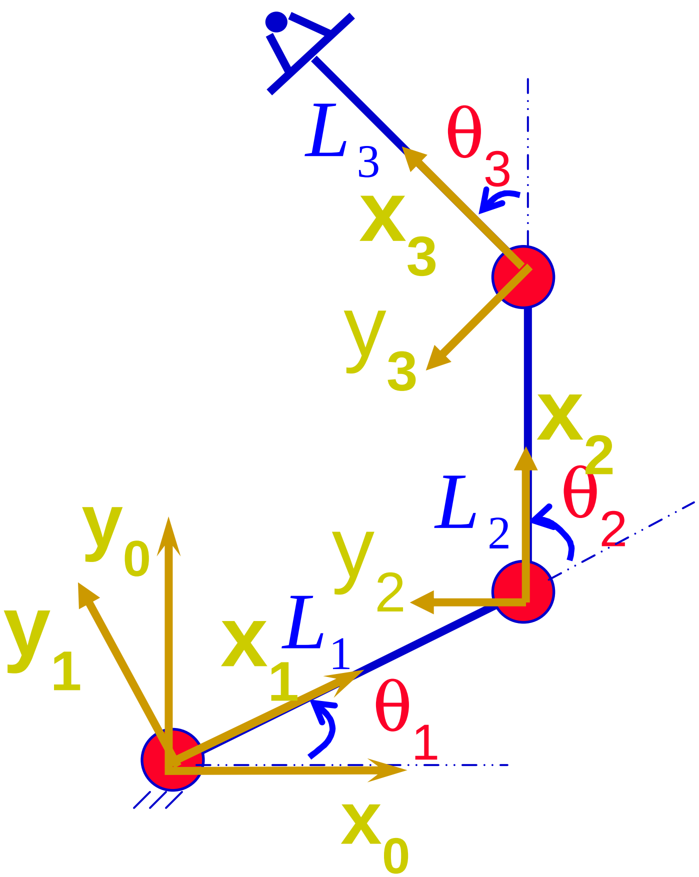
a_i : distance (z_i , z_{i+1}) along x_i

α_i : angle (z_i , z_{i+1}) about x_i

d_i : distance (x_{i-1} , x_i) along z_i

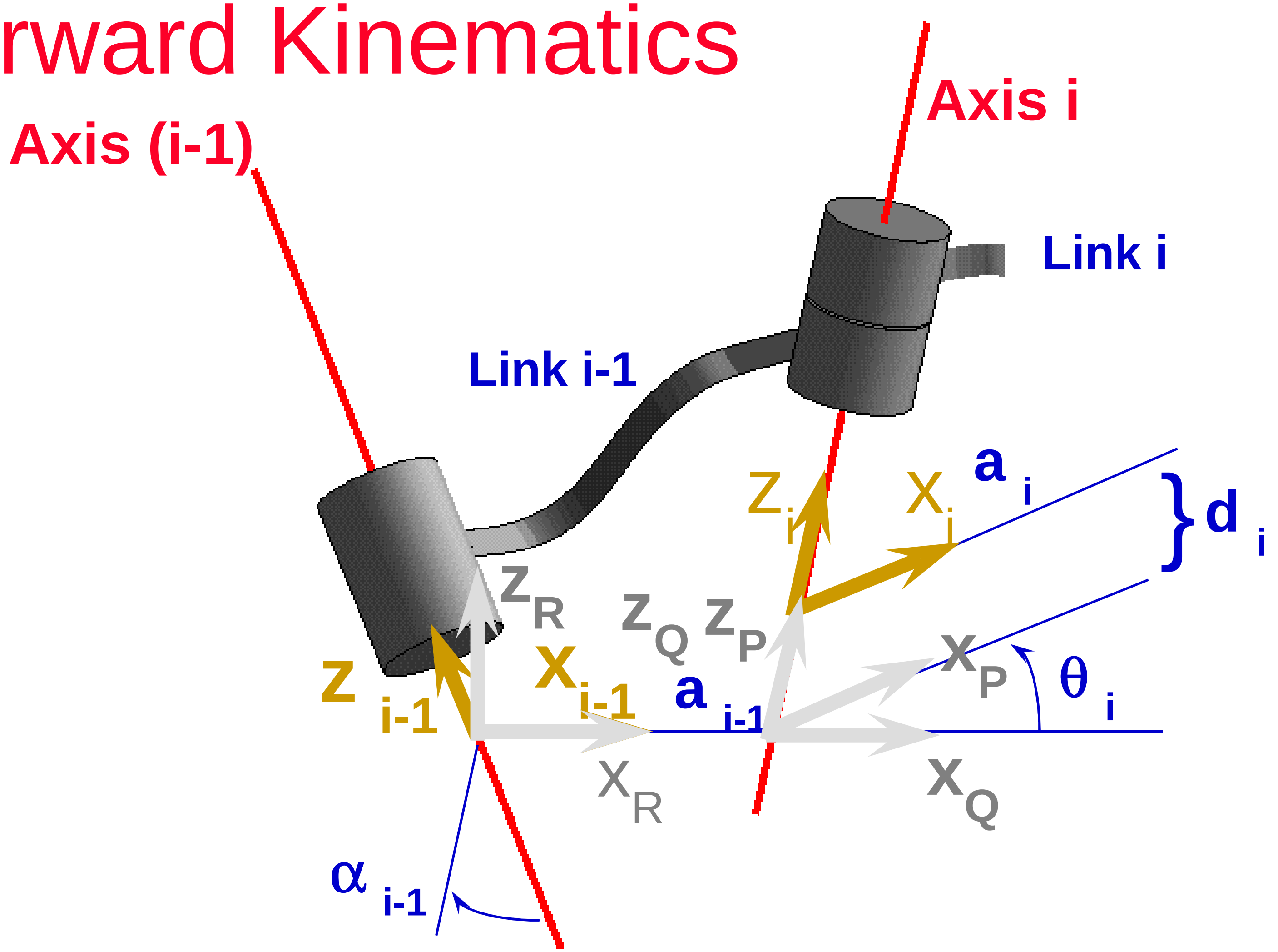
θ_i : angle (x_{i-1} , x_i) about z_i

Example – RRR Arm



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

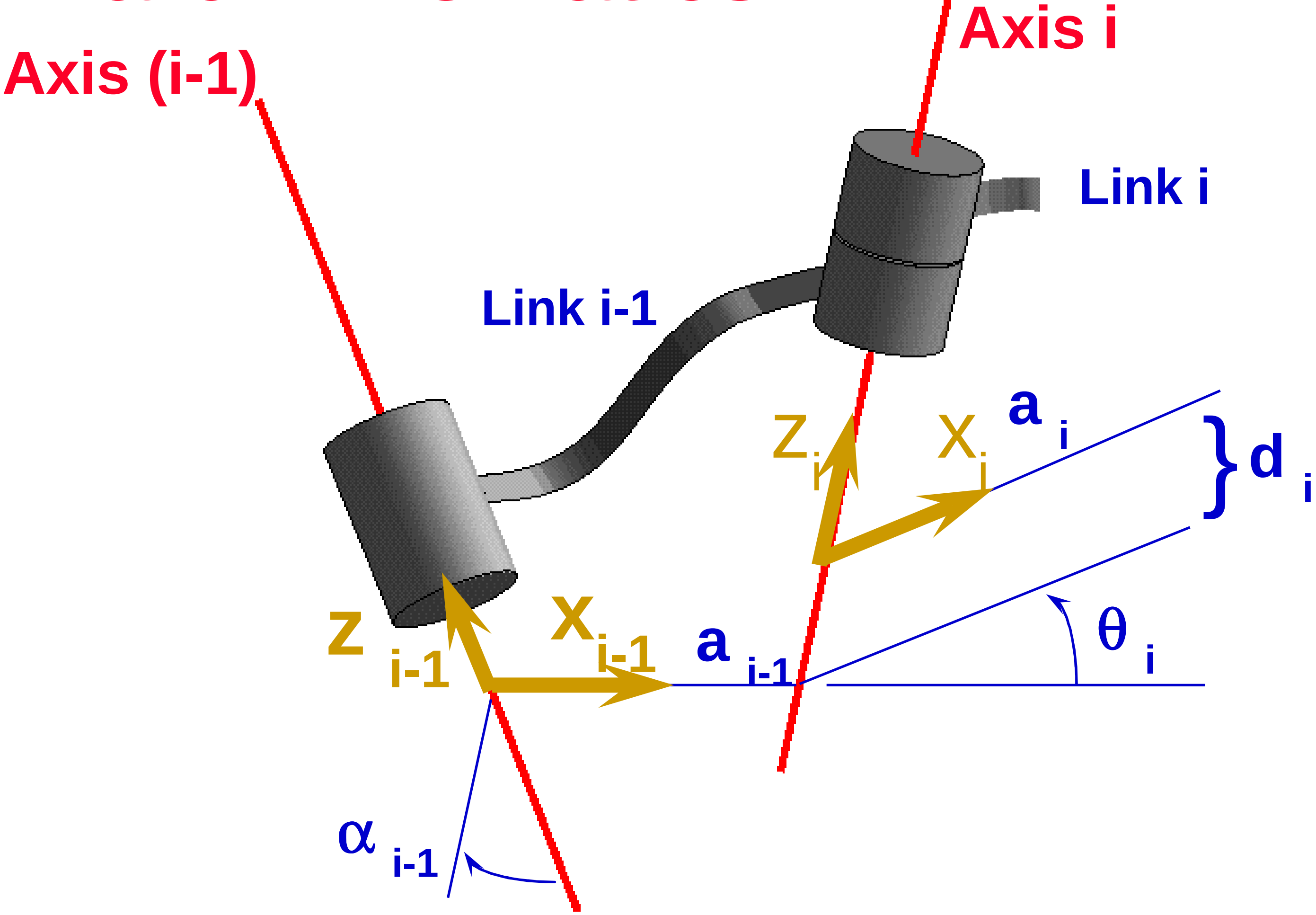
Forward Kinematics



$${}^{i-1}_{\ \ i}T = {}^{i-1}_{\ \ R}T \quad {}^{R}_{Q}T \quad {}^{Q}_{P}T \quad {}^{P}_{i}T$$

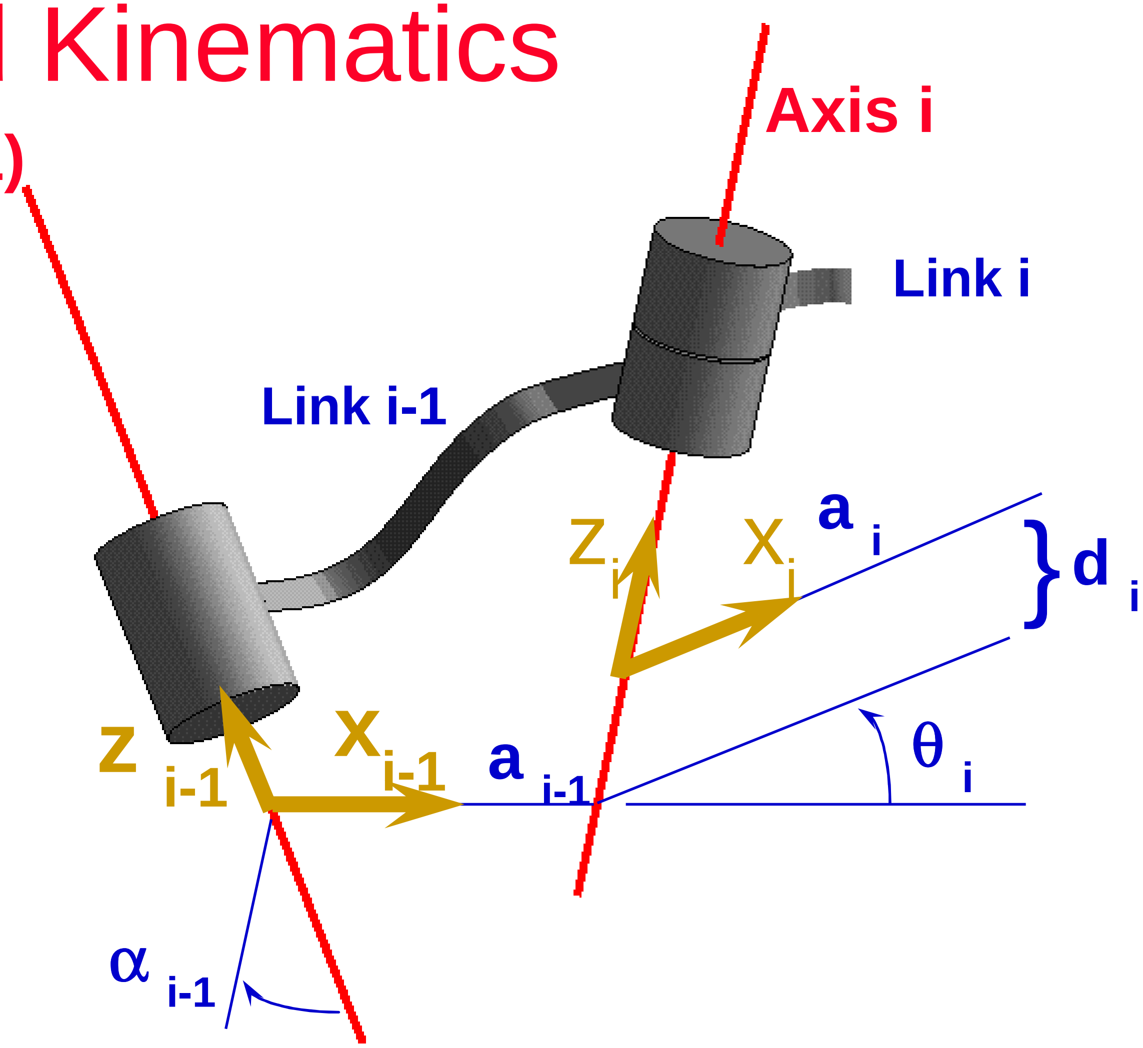
$${}^{i-1}_{\ \ i}T_{(\alpha_{i-1},\, a_{i-1},\, \theta_i,\, d_i)} = R_x(\alpha_{i-1})\, D_x(a_{i-1})\, R_z(\theta_i)\, D_z(d_i)$$

Forward Kinematics



$$
{}^{i-1}_{1}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics



Forward Kinematics: $^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; \ldots \; {}^{N-1}_{N}T$