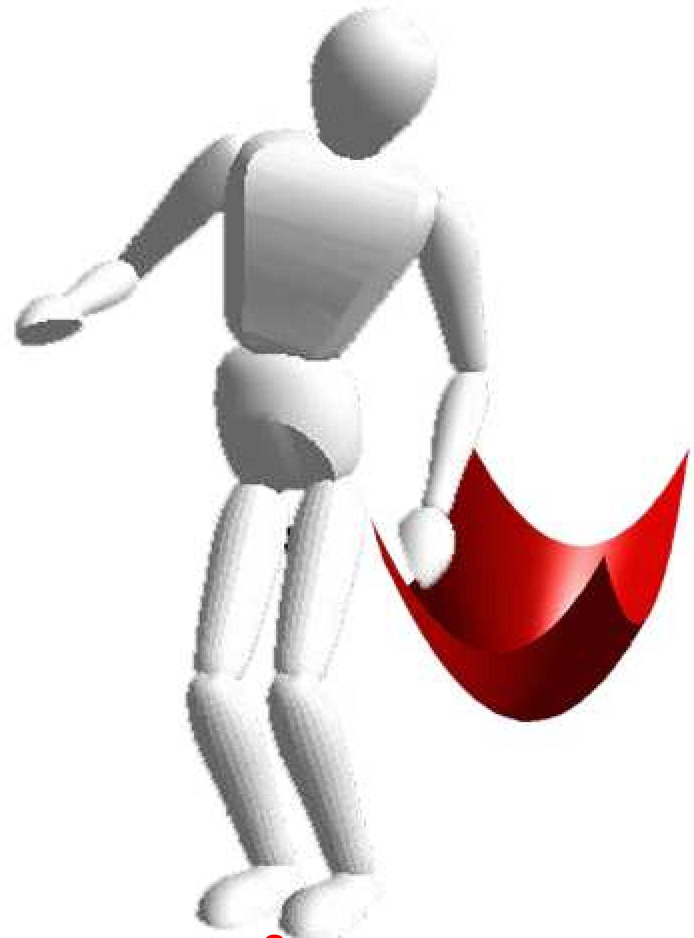
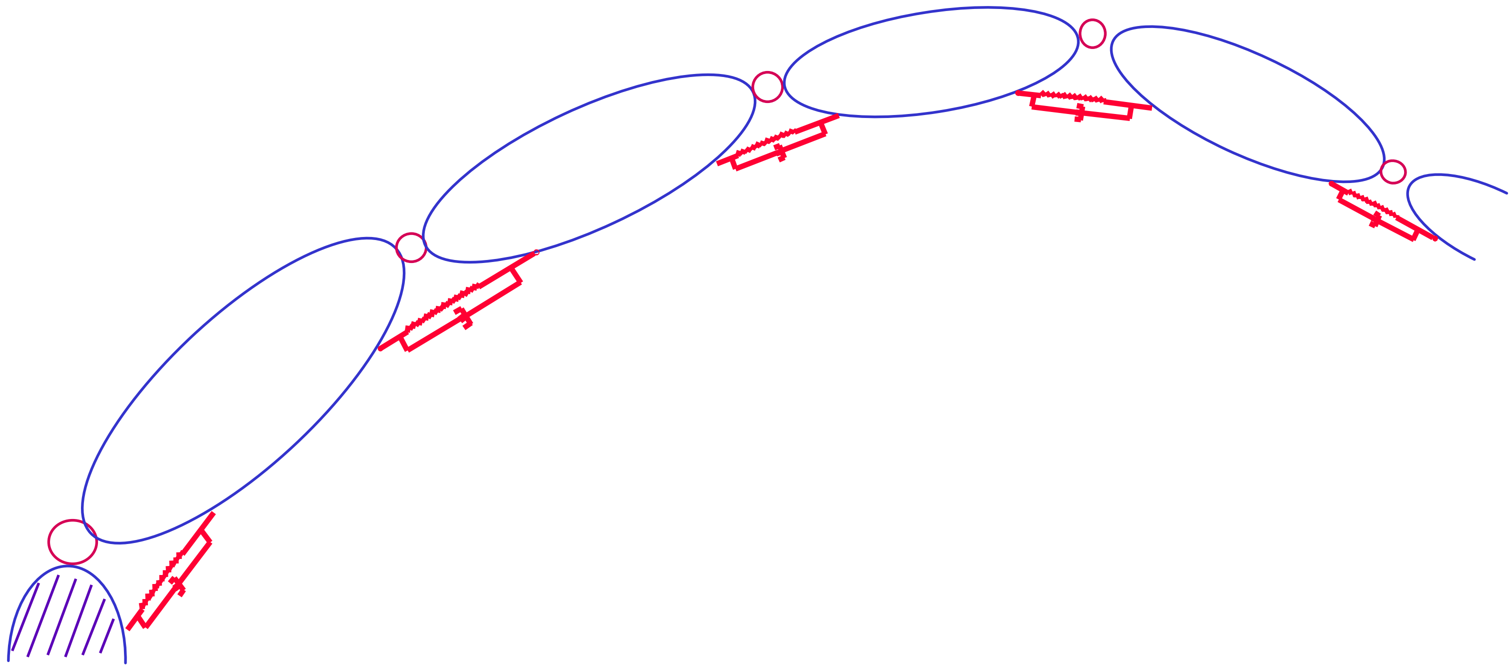


Robot Control

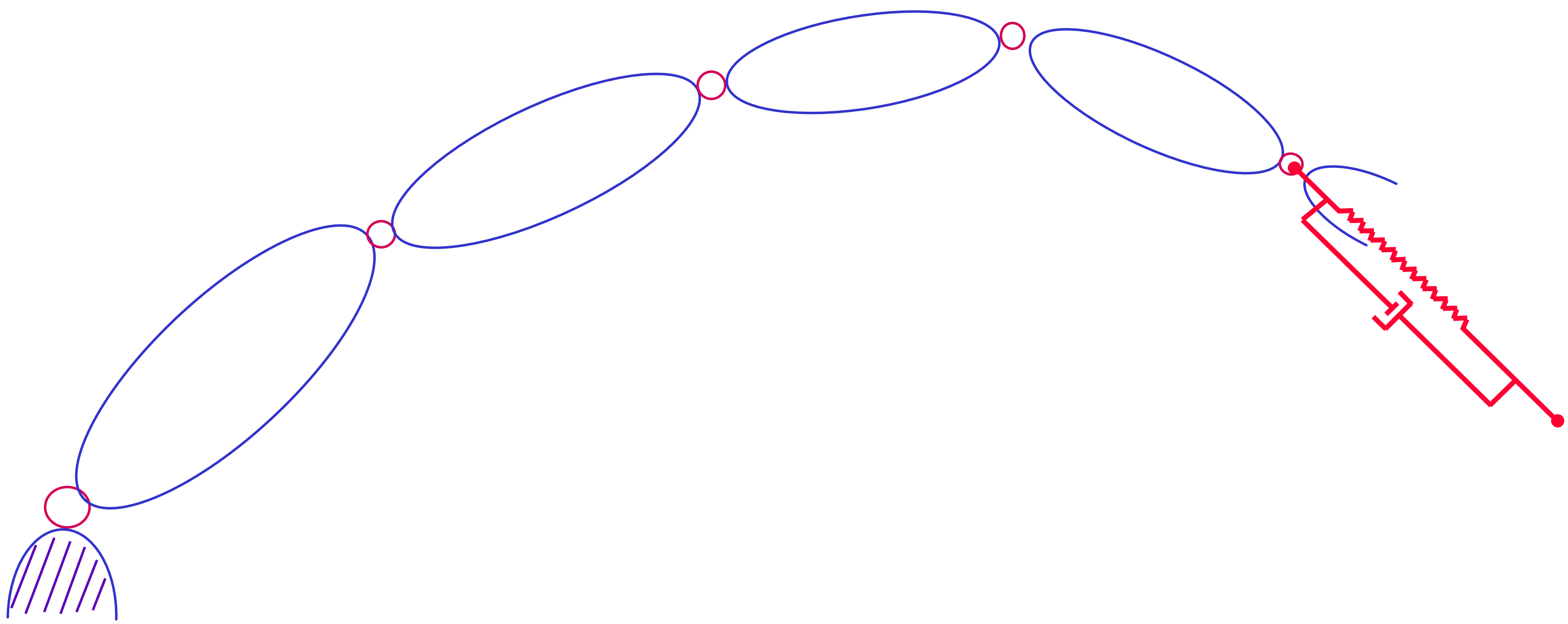
Control

- Natural Systems
- PID Control
- Joint-Space Dynamic Control
- Task-Oriented Control
- Force Control

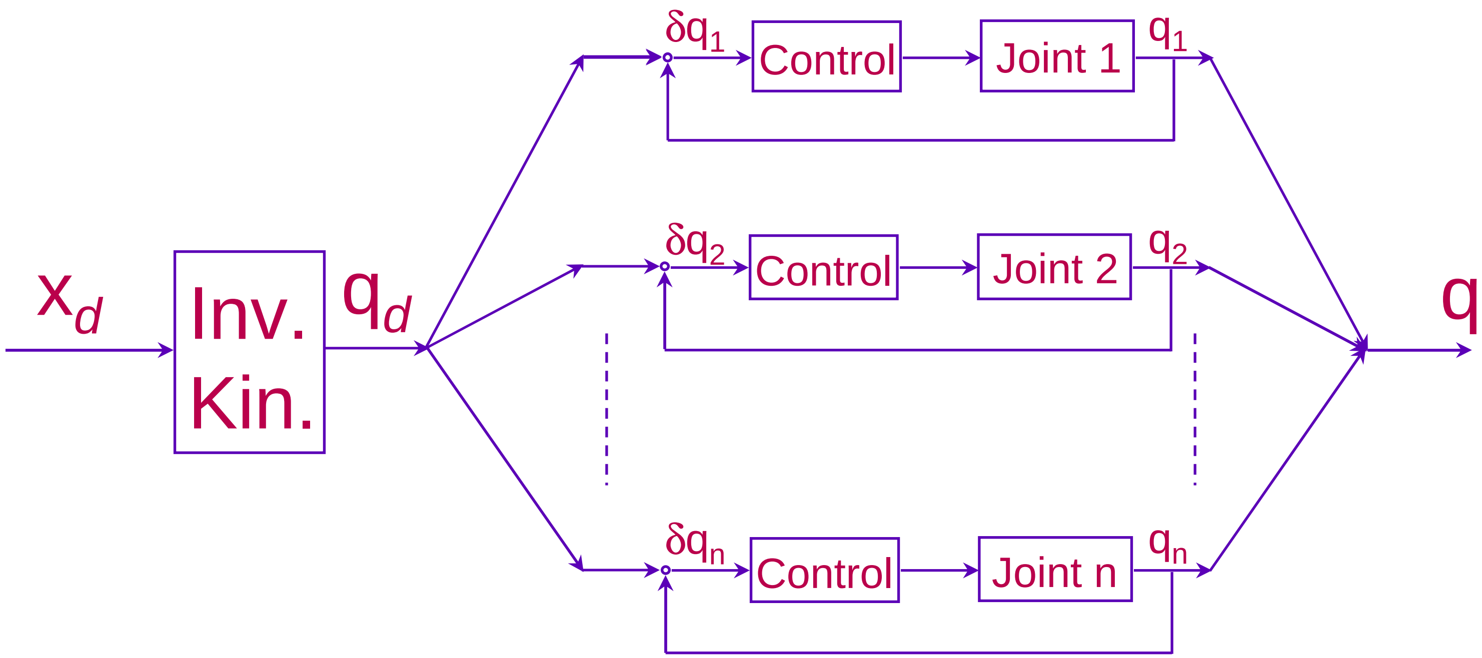
Joint-Space Control



Task-Oriented Control



Joint Space Control



Resolved Motion Rate Control (Whitney 72)

$$\delta x = J(\theta)\delta\theta$$

Outside singularities

$$\delta\theta = J^{-1}(\theta)\delta x$$

Arm at Configuration θ

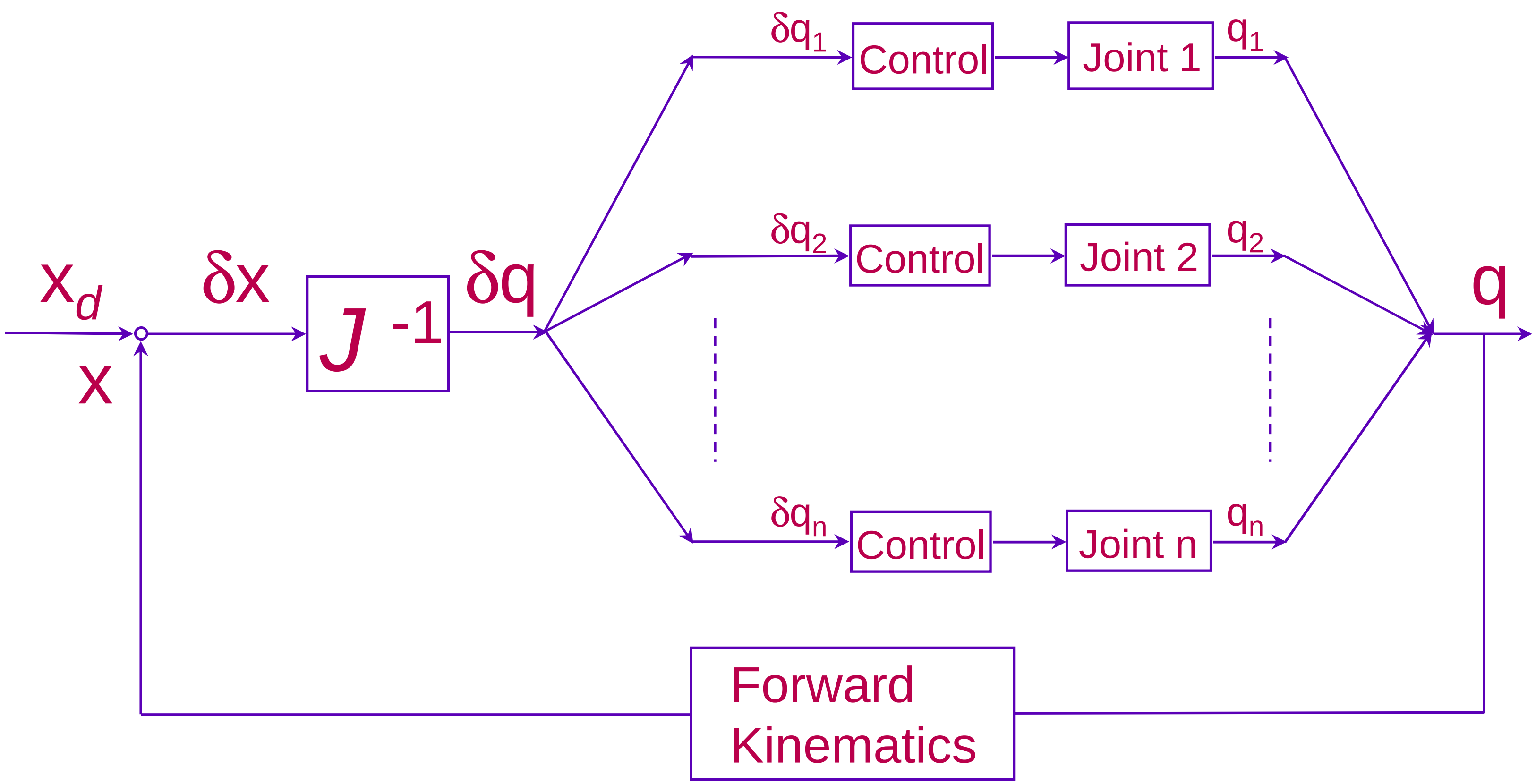
$$x = f(\theta)$$

$$\delta x = x_d - x$$

$$\delta\theta = J^{-1}\delta x$$

$$\boxed{\theta^{+} = \theta + \delta\theta}$$

Resolved Motion Rate Control



Natural Systems

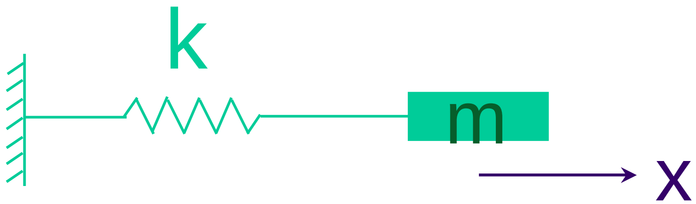
Conservative Systems

$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial \dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

Natural Systems

Conservative Forces



$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right)-\frac{\partial K}{\partial x}=-\frac{\partial V}{\partial x}$$

$$m\ddot{x} = F = -kx$$

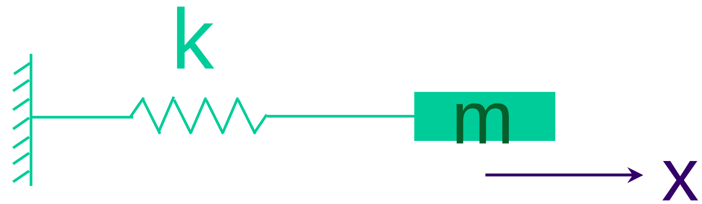
$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Potential Energy of a spring

$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

Natural Systems

Conservative Forces



$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = -\frac{\partial V}{\partial x}$$

Potential Energy of a spring

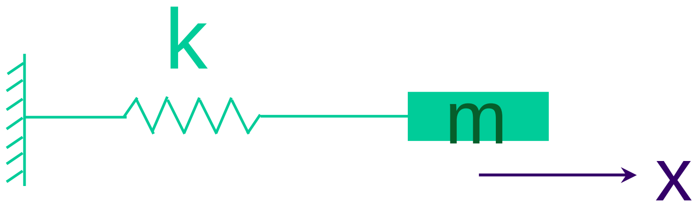
$$V = Work = \int_x^0 (-kx)\delta x = \frac{1}{2}kx^2$$

$$m\ddot{x} = F = -kx$$

$$-\frac{\partial}{\partial x}\left(\frac{1}{2}kx^2\right)$$

Natural Systems

Conservative Systems



$$\frac{d}{dt}\left(\frac{\partial (K-V)}{\partial \dot{x}}\right)-\frac{\partial (K-V)}{\partial x}=0$$

$$K=\frac{1}{2}m\dot{x}^2$$

$$m\ddot{x}+kx=0$$

$$V=\frac{1}{2}kx^2$$

Natural Systems

Conservative Systems

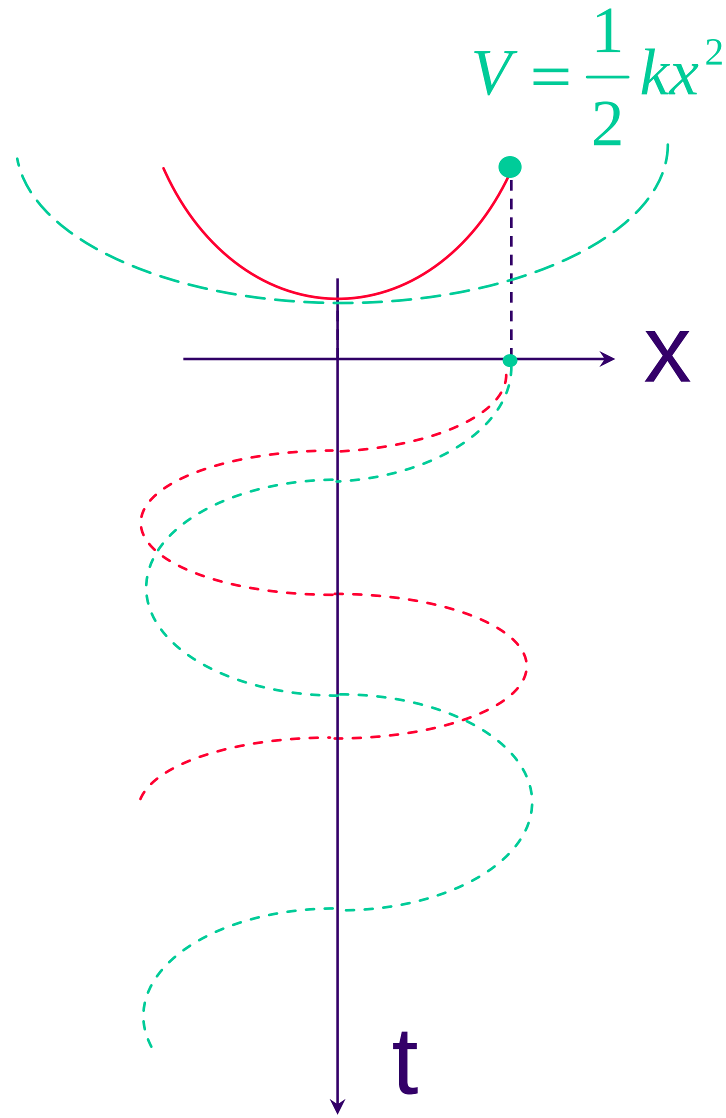
$$m\ddot{x} + kx = 0$$

Frequency increases
with stiffness
and inverse mass

Natural Frequency $\omega_n = \sqrt{\dfrac{k}{m}}$

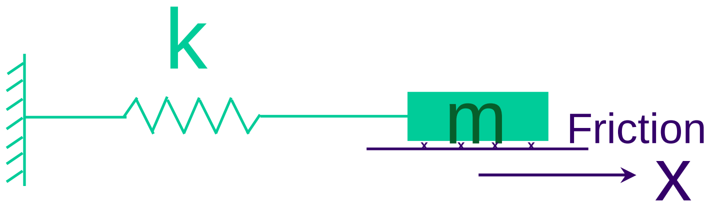
$$\ddot{x} + \omega_n^2 x = 0$$

$$x(t) = c\ \cos(\omega_n t + \phi)$$



Natural Systems

Dissipative Systems

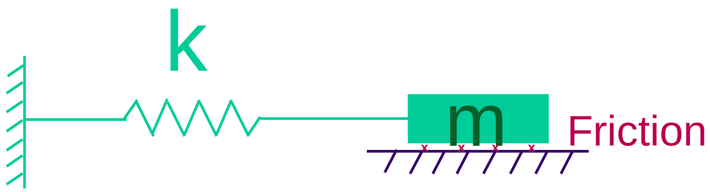


$$\frac{d}{dt}\left(\frac{\partial(K-V)}{\partial\dot{x}}\right)-\frac{\partial(K-V)}{\partial x}=f_{friction}$$

Viscous friction: $f_{friction}=-b\dot{x}$

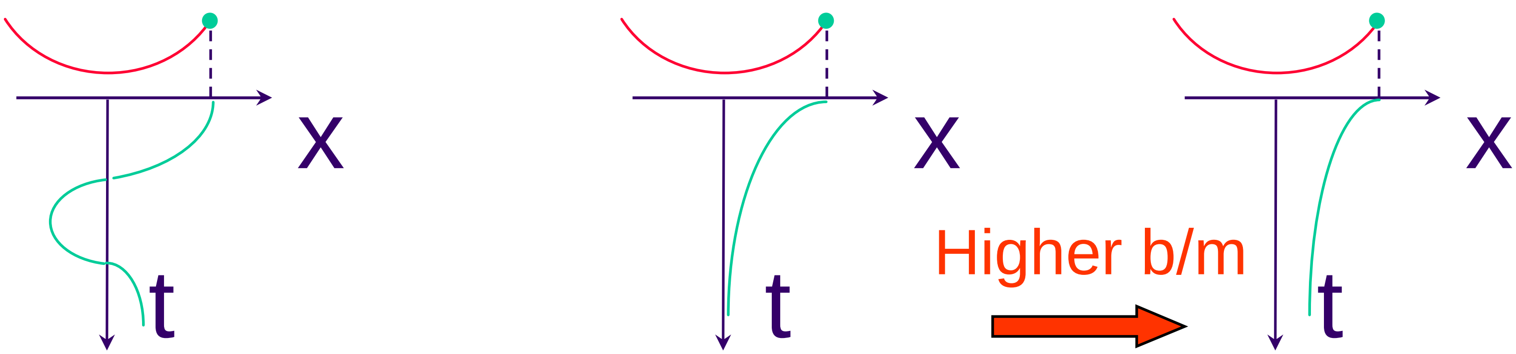
$$m\ddot{x}+b\dot{x}+kx=0$$

Dissipative Systems



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$



Oscillatory damped

Critically damped

$$\frac{b}{m} = 2\omega_n$$

Over damped

2ᵈ order systems

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\frac{b/m}{2\omega_n} \cdot 2\omega_n$$

$$\omega_n^{\,2}$$

Natural damping ratio

Critically damped when b/m=$2\omega_n$

$$\xi_n = \frac{b}{2\omega_n m} = \frac{b}{2\sqrt{km}}$$

Critically damped system: $\xi_n = 1 \ \ (b = 2\sqrt{km})$

Time Response

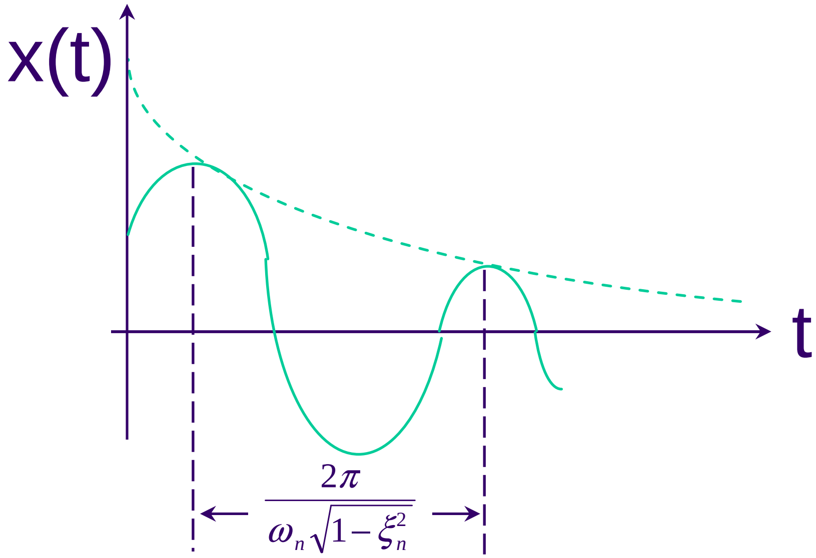
$$\ddot{x} + 2\xi_n \omega_n \dot{x} + \omega_n^2 x = 0$$

Natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} \;;\quad \xi_n = \frac{b}{2\sqrt{km}}$$

Natural damping ratio

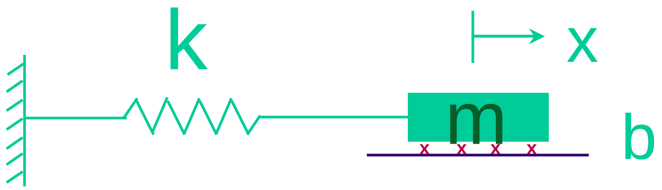
$$x(t) = c e^{-\xi_n \omega_n t} \cos(\underbrace{\omega_n \sqrt{1-\xi_n^2}}_{\omega} t + \phi)$$



damped Natural frequency

$$\omega = \omega_n \sqrt{1-\xi_n^2}$$

Example



$$m\ddot{x} + b\dot{x} + kx = 0$$

$$m = 2.0$$
$$b = 4.8$$
$$k = 8.0$$

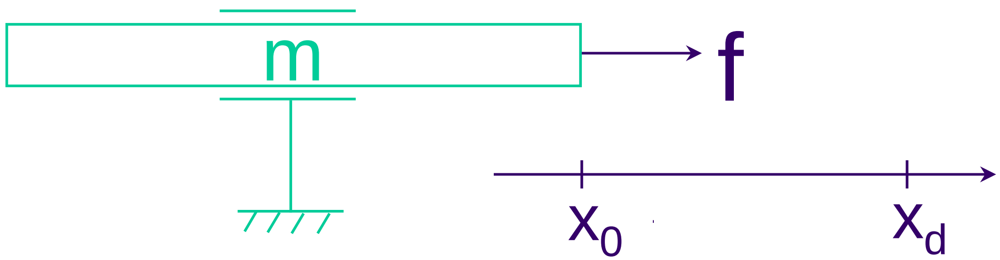
what is the "damped Natural frequency"

$$\omega = \omega_n \sqrt{1 - \xi_n^2}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 2 \; ; \quad \xi_n = \frac{b}{2\sqrt{km}} = 0.6$$

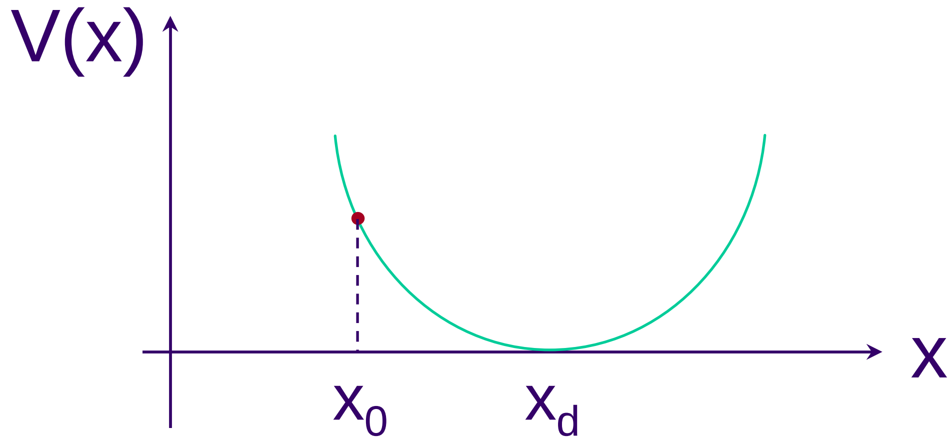
$$\boxed{\omega = 2\sqrt{1 - 0.36} = 1.6}$$

1-dof Robot Control



$$m\ddot{x} = f$$

Potential Field



$$V(x) > 0, x \neq x_d$$

$$V(x) = 0, x = x_d$$

$$V(x) = \frac{1}{2}k_p(x - x_d)^2 \;;\; f = -\nabla V(x) = -\frac{\partial V}{\partial x}$$

$$m\ddot{x} = -\frac{\partial}{\partial x}\left[\frac{1}{2}k_p(x - x_d)^2\right] \;;\; m\ddot{x} + k_p(x - x_d) = 0$$

Position gain

Passive Systems (Stability)

$$V_{goal} = \frac{1}{2} k_p \left(x - x_g\right)^T \left(x - x_g\right)$$

System

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial K}{\partial x} = f$$

$$\Downarrow \quad f = -\frac{\partial V_{goal}}{\partial X}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial \left(K - V_{goal}\right)}{\partial x} = 0$$

Conservative Forces

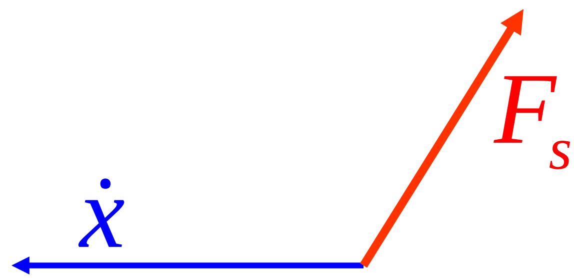
Stable

Asymptotic Stability

a system $$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}}\right) - \frac{\partial\left(K - V_{goal}\right)}{\partial x} = F_s$$

is asymptotically stable if

$$F_s^T \dot{x} < 0 \quad ; \; for \;\; \dot{x} \neq 0$$



$$F_s = -k_v \dot{x} \;\; \rightarrow \;\; k_v > 0$$

Control

$$F = -k_p\left(x - x_{goal}\right) - k_v \dot{x}$$