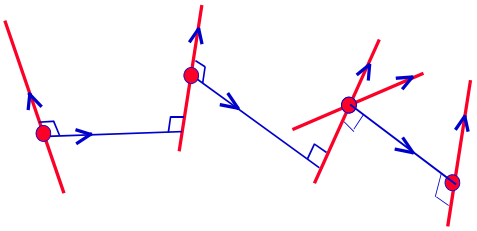
Movie Segment

Learning Locomotion with LittleDog, University of Southern California, 2010

Movie Segment

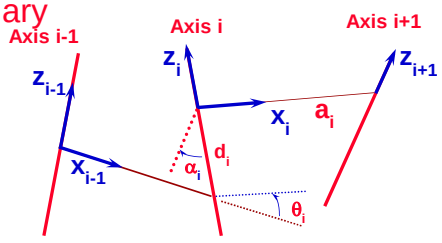
BigDog, Boston Dynamics, 2010

Summary – Frame Attachment



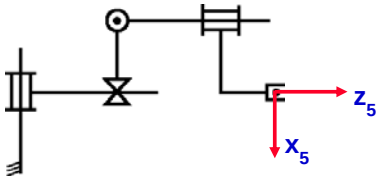
1. Normals
2. Origins
3. Z-axes
4. X-axes

Summary

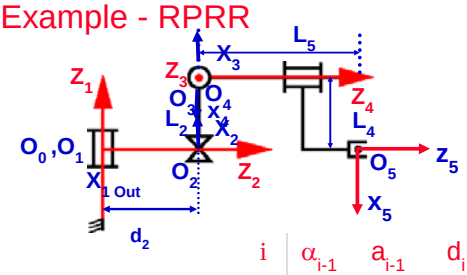


a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i

Example - RPRR



Example - RPRR

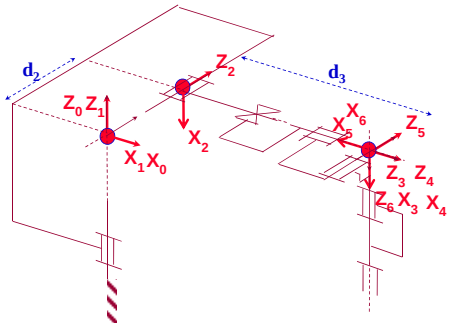


i	α_{i-1}	a_{i-1}	d_i	θ_i
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–

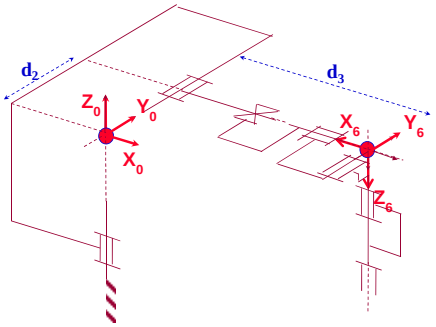
a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
θ_i : angle (x_{i-1} , x_i) about z_i

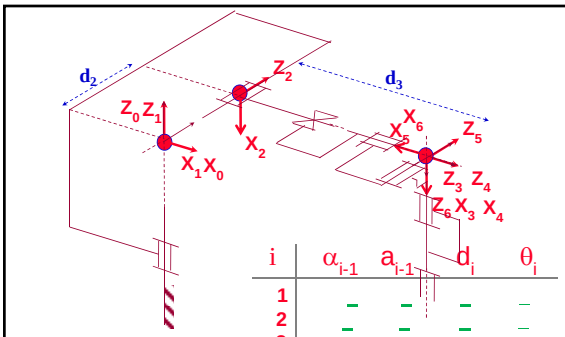
Stanford Scheinman Arm

Stanford Scheinman Arm



Stanford Scheinman Arm





i	α_{i-1}	a_{i-1}	d_i	θ_i
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–
6	–	–	–	–

a_i : distance (z_i , z_{i+1}) along x_i
α_i : angle (z_i , z_{i+1}) about x_i
d_i : distance (x_{i-1} , x_i) along z_i
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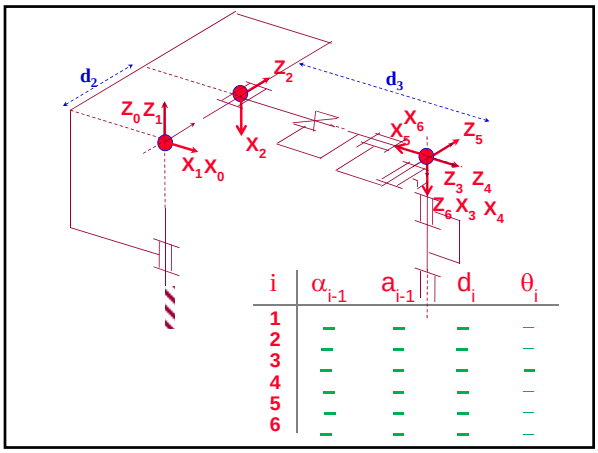
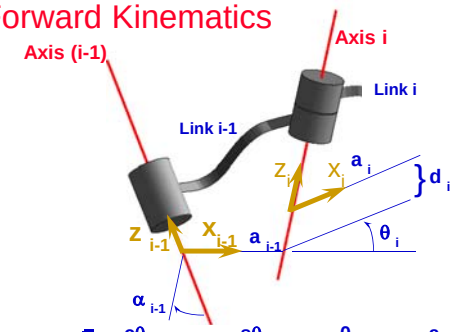


i	α_{i-1}	a_{i-1}	d_i	θ_i
1	–	–	–	–
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–
6	–	–	–	–

Forward Kinematics



$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Stanford Scheinman Arm

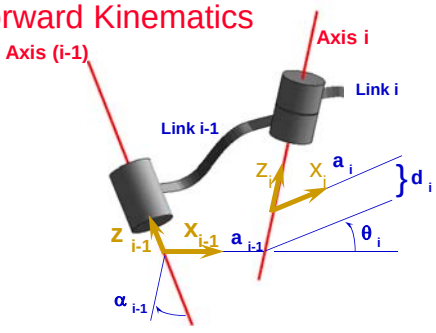
i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2	–	–	–	–
3	–	–	–	–
4	–	–	–	–
5	–	–	–	–
6	–	–	–	–

$$ \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{2}T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{4}_{5}T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{5}_{6}T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Forward Kinematics



Forward Kinematics: $ {}^{0}_{N}T = {}^{0}_{1}T \; {}^{1}_{2}T \; \ldots \; {}^{N-1}_{N}T $

$$ {}^{0}_{1}T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

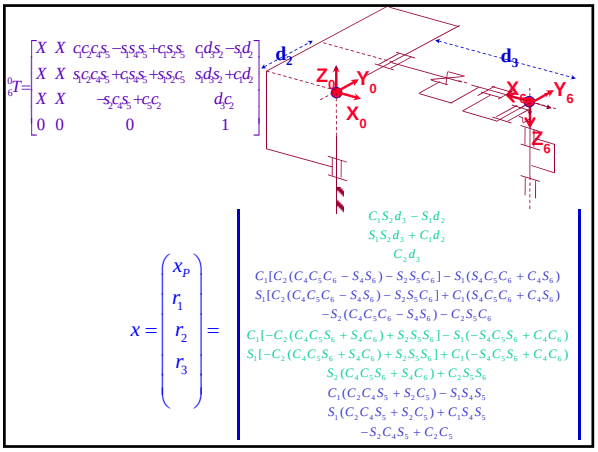
$$ {}^{0}_{2}T = \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 & -s_1d_2 \\ s_1c_2 & -s_1s_2 & c_1 & c_1d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{3}T = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & c_1d_3s_2 - s_1d_2 \\ s_1c_2 & c_1 & s_1s_2 & s_1d_3s_2 + c_1d_2 \\ -s_2 & 0 & c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
{}^{0}_{4}T = \begin{bmatrix}
c_1c_2c_4 - s_1s_4 & -c_1c_2s_4 - s_1c_4 & c_1s_2 & c_1d_3s_2 - s_1d_2 \\
s_1c_2c_4 + c_1s_4 & -s_1c_2s_4 + c_1c_4 & s_1s_2 & s_1d_3s_2 + c_1d_2 \\
-s_2c_4 & s_2s_4 & c_2 & d_3c_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
{}^{0}_{5}T = \begin{bmatrix}
X & X & -c_1c_2s_4 - s_1c_4 & c_1d_3s_2 - s_1d_2 \\
X & X & -s_1c_2s_4 + c_1c_4 & s_1d_3s_2 + c_1d_2 \\
X & X & s_2s_4 & d_3c_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
{}^{0}_{6}T = \begin{bmatrix}
X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\
X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\
X & X & -s_2c_4s_5 + c_5c_2 & d_3c_2 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$



$$
x = \begin{pmatrix} x_P \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} =
\begin{bmatrix}
C_1S_2d_3 - S_1d_2 \\
S_1S_2d_3 + C_1d_2 \\
C_2d_3 \\
C_1[C_2(C_4C_5C_6 - S_4S_6) - S_2S_5C_6] - S_1(S_4C_5C_6 + C_4S_6) \\
S_1[C_2(C_4C_5C_6 - S_4S_6) - S_2S_5C_6] + C_1(S_4C_5C_6 + C_4S_6) \\
-S_2(C_4C_5C_6 - S_4S_6) - C_2S_5C_6 \\
C_1[-C_2(C_4C_5S_6 + S_4C_6) + S_2S_5S_6] - S_1(-S_4C_5S_6 + C_4C_6) \\
S_1[-C_2(C_4C_5S_6 + S_4C_6) + S_2S_5S_6] + C_1(-S_4C_5S_6 + C_4C_6) \\
S_2(C_4C_5S_6 + S_4C_6) + C_2S_5S_6 \\
C_1(C_2C_4S_5 + S_2C_5) - S_1S_4S_5 \\
S_1(C_2C_4S_5 + S_2C_5) + C_1S_4S_5 \\
-S_2C_4S_5 + C_2C_5
\end{bmatrix}
$$