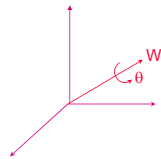


Spatial Descriptions

- Task Description
- Transformations
- Representations ←

Euler Parameters

$$\begin{aligned}\varepsilon_1 &= W_x \cdot \sin \frac{\theta}{2} \\ \varepsilon_2 &= W_y \cdot \sin \frac{\theta}{2} \\ \varepsilon_3 &= W_z \cdot \sin \frac{\theta}{2} \\ \varepsilon_4 &= \cos \frac{\theta}{2}\end{aligned}$$



Normality Condition

$$|\mathbf{W}| = 1, \quad \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$$

ε : point on a unit hypersphere in four-dimensional space

Inverse Problem Given ${}^A_B R$ find ε

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \equiv \begin{bmatrix} 1 - 2\varepsilon_2^2 - 2\varepsilon_3^2 & 2(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2 + \varepsilon_3\varepsilon_4) & 1 - 2\varepsilon_1^2 - 2\varepsilon_3^2 & 2(\varepsilon_2\varepsilon_3 - \varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) & 1 - 2\varepsilon_1^2 - 2\varepsilon_2^2 \end{bmatrix}$$

$$r_{11} + r_{22} + r_{33} = 3 - 4(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) (1 - \varepsilon_4^2)$$

$$\varepsilon_4 = \frac{1}{2} \sqrt{1 + r_{11} + r_{22} + r_{33}}$$

$$\varepsilon_1 = \frac{r_{32} - r_{23}}{4\varepsilon_4}, \quad \varepsilon_2 = \frac{r_{13} - r_{31}}{4\varepsilon_4}, \quad \varepsilon_3 = \frac{r_{21} - r_{12}}{4\varepsilon_4}$$

$$\underline{\underline{\varepsilon_4 = 0?}}$$

Lemma For all rotations one of the Euler Parameters is greater than or equal to 1/2

$$\left(\sum_i \varepsilon_i^2 = 1 \right)$$

Algorithm Solve with respect to $\max_i \{ \varepsilon_i \}$

$$\bullet \varepsilon_1 = \max_i \{ \varepsilon_i \}$$

$$\varepsilon_1 = \frac{1}{2} \sqrt{r_{11} - r_{22} - r_{33} + 1}$$

$$\varepsilon_2 = \frac{(r_{31} + r_{13})}{4\varepsilon_1}, \quad \varepsilon_3 = \frac{(r_{31} + r_{13})}{4\varepsilon_1}, \quad \varepsilon_4 = \frac{(r_{32} - r_{23})}{4\varepsilon_1}$$

$$\bullet \varepsilon_1 = \max_i \{ \varepsilon_i \}$$

$$\varepsilon_1 = \frac{1}{2} \sqrt{r_{11} - r_{22} - r_{33} + 1}$$

$$\bullet \varepsilon_2 = \max_i \{ \varepsilon_i \}$$

$$\varepsilon_2 = \frac{1}{2} \sqrt{-r_{11} + r_{22} - r_{33} + 1}$$

$$\bullet \varepsilon_3 = \max_i \{ \varepsilon_i \}$$

$$\varepsilon_3 = \frac{1}{2} \sqrt{-r_{11} - r_{22} + r_{33} + 1}$$

$$\bullet \varepsilon_4 = \max_i \{ \varepsilon_i \}$$

$$\varepsilon_4 = \frac{1}{2} \sqrt{1 + r_{11} + r_{22} + r_{33}}$$

Euler Parameters / Euler Angles

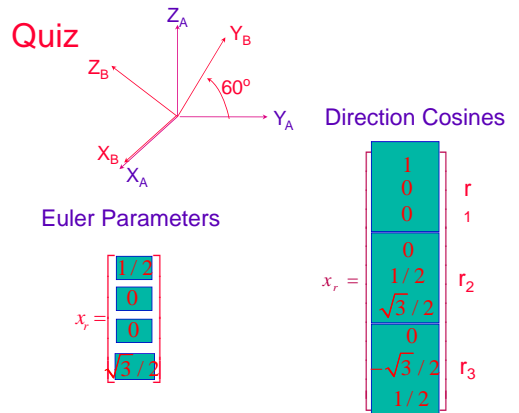
$$\varepsilon_1 = \sin \frac{\beta}{2} \cos \frac{\alpha - \gamma}{2}$$

$$\varepsilon_2 = \sin \frac{\beta}{2} \sin \frac{\alpha - \gamma}{2}$$

$$\varepsilon_3 = \cos \frac{\beta}{2} \sin \frac{\alpha + \gamma}{2}$$

$$\varepsilon_4 = \cos \frac{\beta}{2} \cos \frac{\alpha + \gamma}{2}$$

Quiz



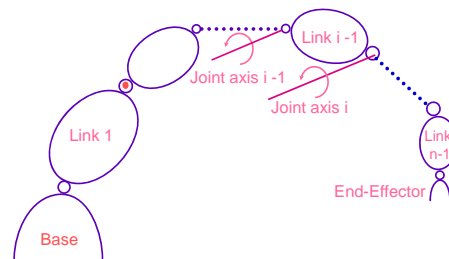
Movie Segment

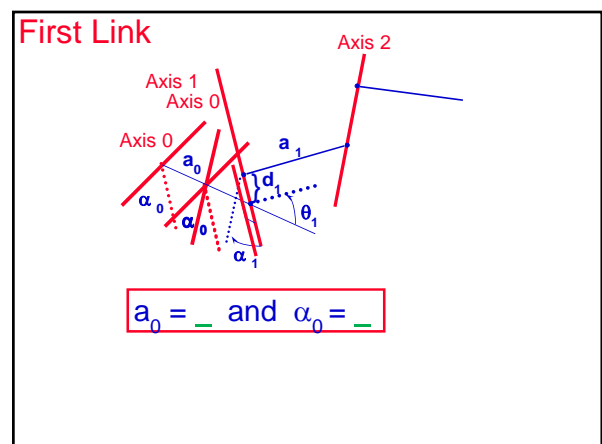
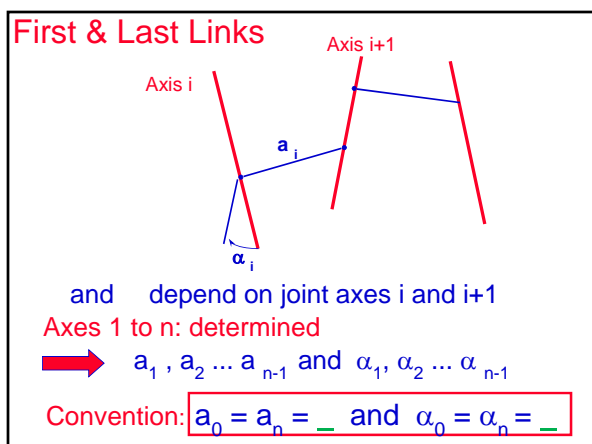
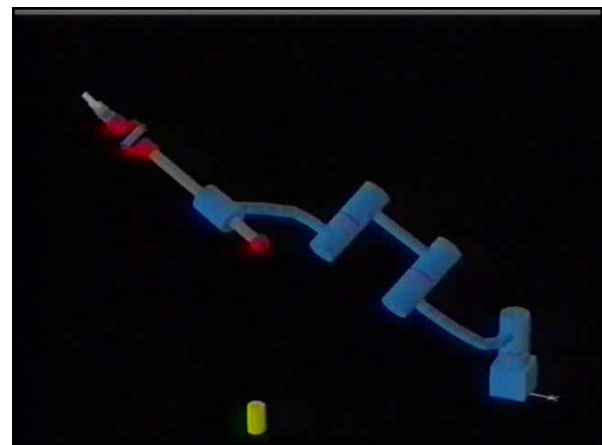
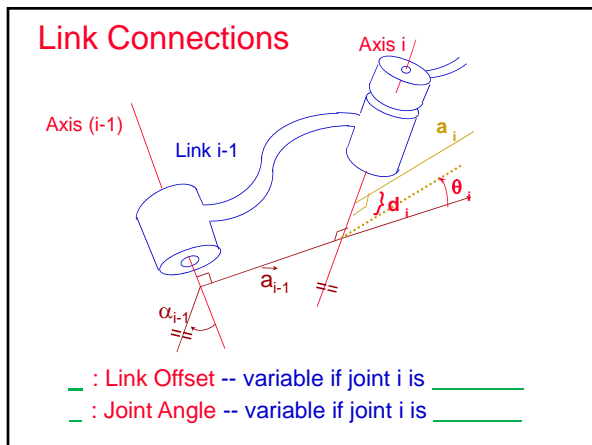
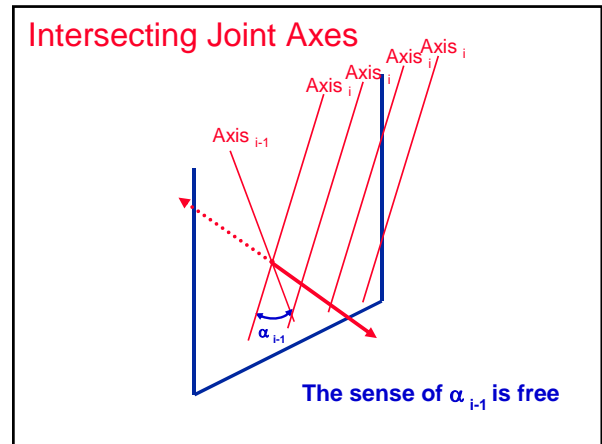
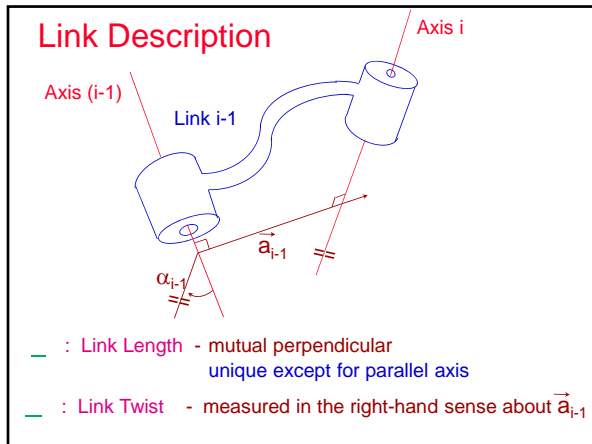
HRP-4, AIST and Kwada Industries, 2010.

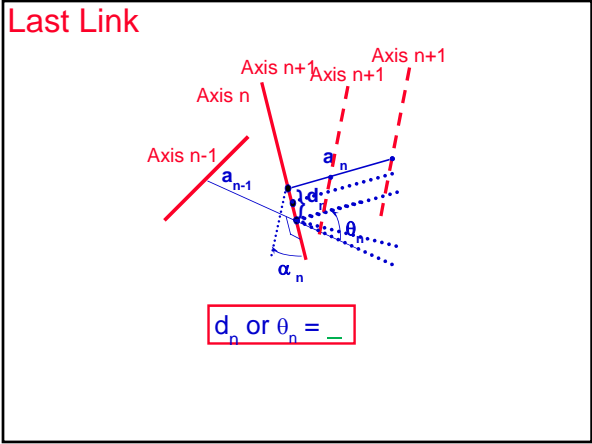
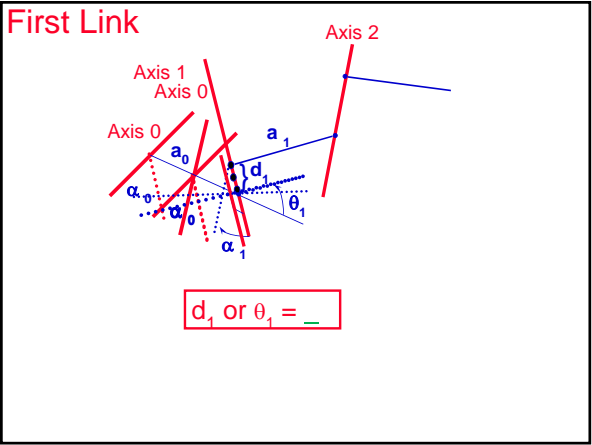
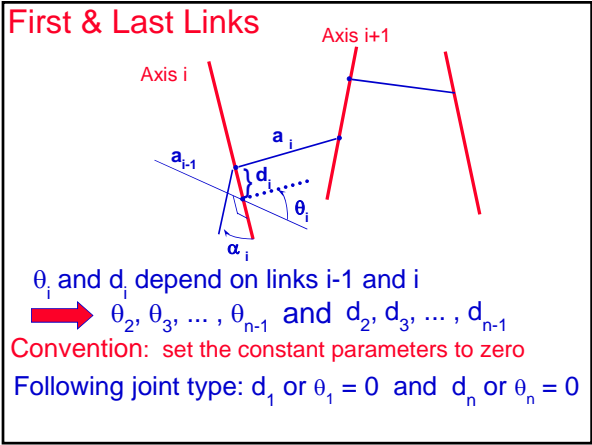
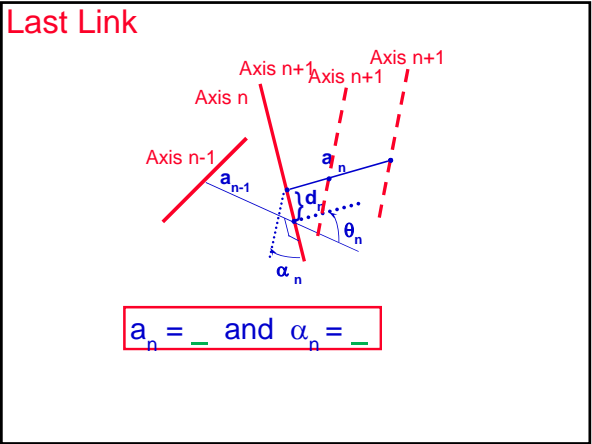
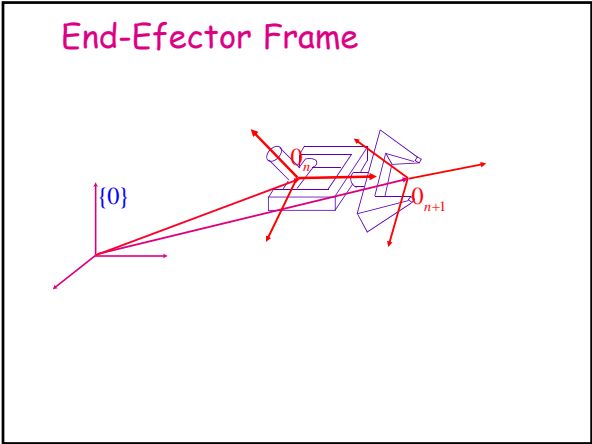
Manipulator Kinematics

- Link Description
- Denavit-Hartenberg Notation
- Frame Attachment
- Forward Kinematics

Manipulator







Denavit-Hartenberg Parameters

4 D-H parameters ($\alpha_i, a_i, d_i, \theta_i$)

3 fixed link parameters

1 joint variable $\left\{ \begin{array}{l} \theta_i \text{ revolute joint} \\ d_i \text{ prismatic joint} \end{array} \right.$

α_i and a_i : describe the Link i
 d_i and θ_i : describe the Link's connection

