Homework #2

CS222 / PHIL358

Due: Monday, May 11

1. In AGM-style belief revision, we can define *expansion*, $K + \phi$, by,

   $$K + \phi = Cn(K \cup \{\phi\}).$$

   Given a particular revision operator $*$, the *contraction* operation can be defined by the *Harper identity*,

   $$K - \phi = K \cap (K * \neg\phi).$$

   Using the AGM postulates for revision, prove the *Levi identity*:

   $$K * \phi = (K - \neg\phi) + \phi.$$

2. Prove the direction $3 \Rightarrow 2$ of Theorem 14.2.2 of *Multiagent Systems*. That is, given a theory K and a revision operator $*$, if $*$ defines a rational consequence relation then $*$ satisfies the AGM postulates.

3. (a) In Elgesem's logic of agency we have an axiom $E\phi \wedge E\psi \rightarrow E(\phi \wedge \psi)$ (where E is read as 'bring it about'). We might consider the converse of this,

   $$E(\phi \wedge \psi) \rightarrow E\phi \wedge E\psi$$

   which says, if the agent can bring about a conjunction, the agent can also bring about each of the conjuncts. Prove that in the resulting logic $\neg E\phi$ is a theorem for any ϕ.

   (b) Is this extra axiom intuitively reasonable if E is interpreted as 'bring it about'? Give a short argument why or why not.

4. (a) In Cohen and Levesque's logic of intentions we can define a notion of competence (about some formula ϕ) by $Competent(\phi) := B(\phi) \rightarrow \phi$. Show that the following formula is then valid:

   $$(PGoal(\phi) \wedge Always(Competent(\phi)) \wedge \neg Before(B(Always(\neg\phi)), \neg G(Later\phi))) \rightarrow Eventually(\phi)$$

   (b) Is this formula still valid is we drop $Always(Competent(\phi))$ from the antecedent? Show why or why not.

5. (**Extra Credit**) In both propositional dynamic logic and in defining common knowledge over an S5 structure we identify certain relations as the reflexive transitive closure of other relations. It should therefore be unsurprising that we obtain similar axiomatizations in these two settings. Show that, in general, if we have some frame (W, R_1, R_2) with two relations, $R_1 = (R_2)^*$ (R_1 is the reflexive transitive closure of R_2) if and only if the following two formulas are both valid on (W, R_1, R_2):

   (i) $\langle 1 \rangle \phi \rightarrow (\phi \vee \langle 1 \rangle(\neg\phi \wedge \langle 2 \rangle\langle 1 \rangle\phi))$

   (ii) $\langle 1 \rangle \phi \leftrightarrow (\phi \vee \langle 2 \rangle\langle 1 \rangle\phi)$