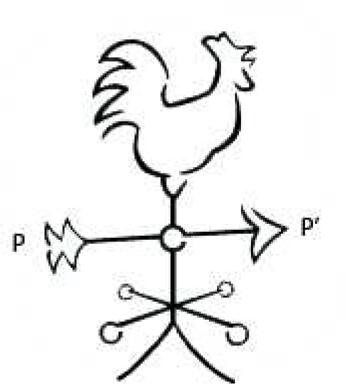
CS 221, Fall 2009 Practice Midterm Solutions: Question 7 Supplement

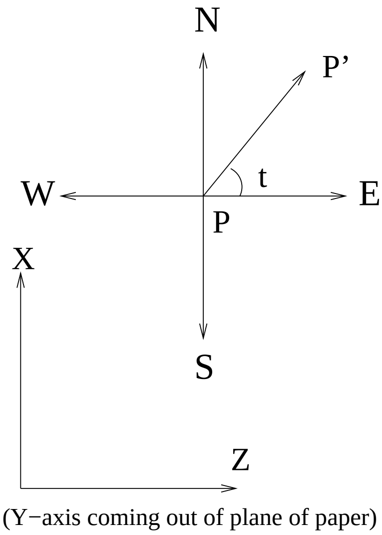
Computer vision question:



Your kite-flying robot needs to know which direction the wind is blowing and has fortuitously found a weathervane with its arrow at known height Y. The robot identifies a point P at the back of the arrow and a point P' at the front of the arrow, both at height Y. Their coordinates as projected onto the robot's image plane are (x, y) and (x', y'), respectively. The robot is facing due east with the Z-axis of its camera perfectly horizontal and at height 0. The X-axis is pointing due north. The focal length of the camera is known to be f. The length of the vane is unknown.

In what direction is the wind blowing? (Assume that the wind is blowing in the direction from P to P'.) Report your answer as an expression for the angle between due east and the wind. I.e., report the angle t as pictured in the diagram below, as a function of x, y, x', y', Y and f.

(**Hint:** Because the height Y of the vane is known, you should be able to find the 3-D coordinates of points P and P'.)



Disclaimer: This is an explanation of the problem and the solution, and is definitely much longer than what you would be expected to write on the midterm. The reference solution for this problem is only a few lines long.

Detailed explanation:

Consider a robot (with a camera) standing on the ground, facing East. The X, Y and Z axes of the world are defined as follows:

- The Z axis goes through the center of the camera (so it points in the direction the robot is facing, or East)
- The X axis points North
- The Y axis points up towards the sky.

Nearby is a weathervane at some height Y from the ground. It is defined by its two endpoints points, P and P', which have coordinates (X, Y, Z) and (X', Y, Z'). Note that we are assuming that the weathervane is parallel to the ground, so the Y coordinate of its two endpoints is the same.

To figure out which direction the weathervane is pointing, or to compute the angle t defined in the picture above (so the angle that the weathervane makes with the East, or Z, axis in the North, or X, direction), we use the formula

$$t = \tan^{-1}\left(\frac{X' - X}{Z' - Z}\right) \tag{1}$$

The height off the ground plays no role, since we are just interested in computing the direction. You have to be careful to stick to the definition of t that we provide in the problem and to not reverse the X and Z axes in the equation above (and not to introduce the Y axis into the equation). So far we have done nothing computer vision-related, this is just geometry.

Now the robot takes a picture of this weathervane, and instead of the points P = (X, Y, Z) and P' = (X', Y, Z'), it just sees their projection onto the camera plane: (x, y) and (x', y'). Refer to Lecture Notes 10, page 2, for the perspective camera equations, and verify that the projections of the two points will be precisely defined as:

$$\texttt{Point P:} \qquad x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z} \tag{2}$$

$$\texttt{Point P':} \qquad x' = f\frac{X'}{Z'} \qquad y' = f\frac{Y}{Z'} \tag{3}$$

Recall that we are interested in computing the angle t, so we care about the variables X, X', Z and Z'. So let's define each of these variables in terms of Y. From the equation for y of point P in (2), we get

$$Z = f\frac{Y}{y} \tag{4}$$

From the equation for x of point P in (2), and plugging in (4), we get

$$X = \frac{xZ}{f} = \frac{x\left(f\frac{Y}{y}\right)}{f} = \frac{Yx}{y} \tag{5}$$

Similarly, we repeat the process for point P' and get

$$Z' = f\frac{Y}{y'} \qquad X' = \frac{Yx'}{y'} \tag{6}$$

Now we are ready to substitute everything back into our equation for the angle t:

$$t = \tan^{-1}\left(\frac{X' - X}{Z' - Z}\right) = \tan^{-1}\left(\frac{\left(\frac{Yx'}{y'}\right) - \left(\frac{Yx}{y}\right)}{\left(f\frac{Y}{y'}\right) - \left(f\frac{Y}{y}\right)}\right) \tag{7}$$

As a last step, we divide through by Y to simplify the solution. This is the only simplification that's really necessary; you can also rearrange the other terms if you want, but you definitely don't have to and probably shouldn't spend time doing it on the midterm.

$$t = \tan^{-1}\left(\frac{x'/y' - x/y}{f/y' - f/y}\right) \tag{8}$$

Note that the final answer does not depend on how far off the ground the weathervane is (the Y variable), nor does it explicitly require the length of the weathervane.

Good luck!

Reference solution (as in the practice midterm solutions handout):

From the perspective camera equations, if the physical coordinates of points P and P' are (x, y) and (x', y') respectively, we get:

$$Z = fY/y, \quad X = xZ/f = xY/y.$$
$$Z' = fY/y', \quad X' = x'Z/f = x'Y/y'.$$

The angle t is the angle that the line from (X, Y, Z) to (X', Y, Z') makes with the Z-axis:

$$t = \tan^{-1}(\frac{X' - X}{Z' - Z}) = \tan^{-1}(\frac{x'/y' - x/y}{f/y' - f/y})$$