<u>CS205 – Class 11</u>

Covered in class: Everything
Readings: Shewchuk Paper on course web page and Heath 473-478

1. **<u>Conjugate Gradient Method</u>** – this covers more than just optimization, e.g. we'll use it later as an iterative solver to aid in solving PDEs
2. Conjugate Directions
    a. The goal is to choose a sequence of search directions s_0, s_1, … that are all orthogonal. Then only one step is needed in each search direction
    b. We update $x_{k+1} = x_k + s_k \alpha_k$ where each α_k is chosen so that all the remaining error is orthogonal to the search direction s_k . That is, $e_{k+1} \cdot s_k = 0$ and we never need to step in the s_k direction again.
        i. Note that $x_{k+1} = x_k + s_k \alpha_k$ leads to $x_{k+1} - x_{exact} = x_k - x_{exact} + s_k \alpha_k$ or $e_{k+1} = e_k + s_k \alpha_k$ (we'll use this later too!)
        ii. $e_{k+1} \cdot s_k = 0$ leads to $\left(e_k + s_k \alpha_k\right) \cdot s_k = 0$ or $\alpha_k = -\dfrac{e_k \cdot s_k}{s_k \cdot s_k}$
            1. however we don't know e_k
    c. Just as before, instead of using e, we can use r=-Ae to obtain $Ae_{k+1} \cdot s_k = 0$ or $r_{k+1} \cdot s_k = 0$
        i. This leads to $A\left(e_k + s_k \alpha_k\right) \cdot s_k = 0$ or $\alpha_k = -\dfrac{s_k \cdot Ae_k}{s_k \cdot As_k} = \dfrac{s_k \cdot r_k}{s_k \cdot As_k}$
        ii. When $s_k = r_k$ this is the steepest decent method
    d. The only issue now is how to choose the s_k , and the trick of conjugate directions is to choose them *A-orthogonal* instead of orthogonal
        i. That is, $s_j \cdot As_k = 0$ for $j \neq k$ (instead of $s_j \cdot s_k = 0$)
        ii. If A is symmetric, $0^T = \left(s_j \cdot As_k\right)^T = \left(s_j^T As_k\right)^T = s_k^T As_j = s_k \cdot As_j$ and the A-orthogonal relationship is symmetric