Problem 2-1.5. $T(n) = \sqrt{n}T(\sqrt{n}) + n$.
Solution: We prove $T(n) \geq c_1 n \log\log n$ for $c_1 = 1/2$ by substitution. Let $T(\sqrt{n}) \geq c_1\sqrt{n}\log\log\sqrt{n}$. Then,

$$\begin{aligned} T(n) &\geq \sqrt{n} \cdot c_1\sqrt{n}\log\log\sqrt{n} + n \\ &= n(c_1 \log\log n - c_1 \log 2 + 1) \\ &\geq n \log\log n \end{aligned}$$

for $c_1 = 1/2$ assuming log is to the base 2.

Similarly, we prove $T(n) \leq c_2 n \log\log n$ for $c_2 = 2$ by substitution. Let $T(\sqrt{n}) \leq c_2\sqrt{n}\log\log\sqrt{n}$. Then,

$$\begin{aligned} T(n) &\leq \sqrt{n} \cdot c_2\sqrt{n}\log\log\sqrt{n} + n \\ &= n(c_2 \log\log n - c_2 \log 2 + 1) \\ &\leq n \log\log n \end{aligned}$$

for $c_2 = 2$ assuming log is to the base 2.

Concludingly, since $c_1 n \log\log n \leq T(n) \leq c_2 n \log\log n$, $T(n) = \Theta(n \log\log n)$.