CS 154	Intro. to Automata and Complexity Theory	Handout 10
Autumn 2009	David Dill	October 13, 2009

Problem Set 3 Due: October 20, 2009

Homework: (Total 100 points) Do the following exercises.

Problem 1. [30 points] Give an algorithm to solve the following decision problems. In each case you can assume that you are given a concrete representation of the language in the form of a DFA. You must justify the correctness of the algorithm to receive full credit.
a). Suppose L is a regular language over the alphabet $\Sigma = \{0, 1\}$. Give an algorithm to decide whether $|L| \geq 100$.
b). Suppose L is a regular language over the alphabet $\Sigma = \{0, 1\}$. Give an algorithm to decide whether L contains *any* string w such that w has zero occurrences of the patterns 00 and 11.

Problem 2. [10 points] Show how to modify the product construction for DFAs, discussed in class, such that the resulting DFA accepts the union, as opposed to the intersection, of the languages of the two given DFAs. Prove that your construction is correct. (*Hint*: Refer to the discussion of the product construction in the textbook, on pages 136-138 of the third edition, or page 135 of the second edition.)

Problem 3. [20 points] Consider the DFA given by the following transition table.

	0	1
$\rightarrow A$	B	J
B	H	C
$*C$	D	G
D	E	C
$*E$	F	G
F	C	E
$*G$	J	C
H	B	G
I	H	E
J	H	A

Give the minimum equivalent DFA. For each state of the minimized DFA, specify the set of equivalent states of the original DFA.

For these questions, acceptance of a string by a Turing machine is considered to be acceptance by final state. The execution trace of a Turing machine on an input string is the complete sequence of instantaneous descriptions (separated by the $\vdash$ symbol to indicate valid moves of the Turing machine) that the Turing machine goes through when started with that input string on the tape.

Problem 4. [20 points] You are required to construct a (deterministic) Turing machine which takes as input a number n in binary and subtracts 1 from it. The tape initially contains the symbol $ followed by the number n in binary. The tape head is initially scanning the rightmost bit of n. Your machine should halt with the binary number $n-1$ on the tape and the head scanning the rightmost bit again. It is also required that you replace any leading zeroes by the $ symbol. We give below the initial and final ID's for some sample inputs using the inital state q_0 and halting state q_f.

$\$1001q_0 1 \overset{*}{\vdash} \$1001q_f 0$

$\$1000q_0 0 \overset{*}{\vdash} \$\$111q_f 1$

You may assume that $n \geq 2$, and that the input string is always in the specified format and does not contain any leading zeroes.

(a) [15 points] Give the transitions for this Turing machine and explain the purpose of each state.

(b) [5 points] Provide a sample execution trace of your machine for the input $1010.

Problem 5. [20 points] Suppose that we make the following changes to the definition of Turing machines. For each change, characterize the class of languages that can be accepted by the modified Turing machines, and provide a brief justification for your answer.

(a) The Turing machine is prohibited from writing to the tape the symbol that it reads on a transition (i.e., transitions of the form $\delta(q, X) = (p, X, D)$ are not allowed).

(b) The Turing machine can move its head two cells on a transition. Specifically, the transition function now has the form

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \Gamma \times \{L_1, L_2, R_1, R_2\}.$$

A transition includes two tape symbols, the second of which has no effect in the case of a standard move to the left or right (L_1 or R_1). A two-cell transition, which is denoted by L_2 or R_2, changes the ID as follows.

If $\delta(q, X_i) = (p, Y_1, Y_2, L_2)$:

$$X_1 X_2 \cdots X_{i-3} X_{i-2} X_{i-1} q X_i X_{i+1} \cdots X_n \vdash X_1 X_2 \cdots X_{i-3} p X_{i-2} Y_2 Y_1 X_{i+1} \cdots X_n$$

If $\delta(q, X_i) = (p, Y_1, Y_2, R_2)$:

$$X_1 X_2 \cdots X_{i-1} q X_i X_{i+1} X_{i+2} \cdots X_n \vdash X_1 X_2 \cdots X_{i-1} Y_1 Y_2 p X_{i+2} \cdots X_n$$