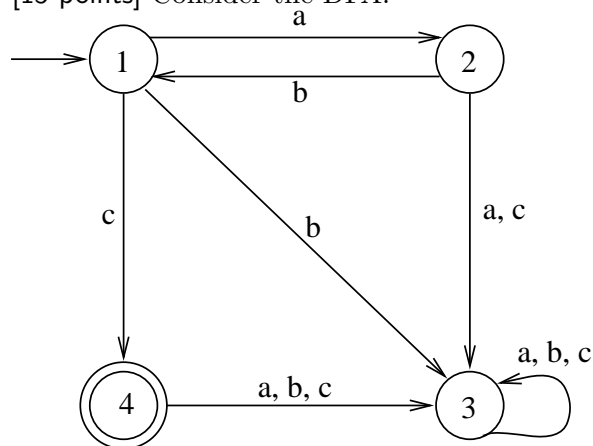


Problem Set 2 Due: October 13, 2009

Homework: (Total 100 points) Do the following exercises.

Problem 1. [15 points] Consider the DFA:



Using the state elimination procedure described in lecture, find a regular expression for the language of this automaton. Draw the GNFA's resulting from each step: (1) a GNFA equivalent to this DFA, (2) eliminate state 3, (3) elimination state 2, (4) eliminate state 1, (5) eliminate state 4.

Problem 2. [40 points] Show that the two languages below are not regular. For each language, give *two separate proofs*, one using the Pumping Lemma, and (2) one using closure properties of the regular languages discussed in the textbook and lecture, and the fact that the language $\{0^i 1^i \mid i \geq 0\}$ is not regular, and *not* the Pumping Lemma.

a). [20 points] The language L_1 over $\Sigma = \{0, 1\}$ consisting of all strings where the number of 1's is exactly two times the number of 0's. (**Hint:** For the closure properties proof, using set intersection and inverse homomorphism.)

b). [20 points] The language $L_2 = \{0^n 1^m 2^{n-m} \mid n \geq m \geq 0\}$ over $\Sigma = \{0, 1, 2\}$.

Problem 3. [15 points] (This is exercise 4.1.2(b) in both the second and third editions of the textbook.)

Prove that $\{0^n \mid n \text{ is a perfect cube}\}$ is not a regular language. (*Hint:* See the solution for Exercise 4.1.2(a).)

Problem 4. [15 points] (Exercise 4.2.6(c) in both editions of the textbook.) Show that the regular languages are closed under the following operation:

$$\mathit{init}(L) = \{w \mid \text{for some string } x, wx \text{ is in } L\}.$$

Hint: It is easiest to start with a DFA for L and perform a construction to get the desired language.

Problem 5. [15 points] (Exercise 4.2.1(f) in both editions of the textbook.) Suppose h is the homomorphism from the alphabet $\{0, 1, 2\}$ to the alphabet $\{a, b\}$ defined by: $h(0) = a$; $h(1) = ab$, and $h(2) = ba$. If L is the language $L(\mathbf{a(ba)^*})$, what is $h^{-1}(L)$? Provide a proof to justify your claim.