Problem Set 1 Due: October 6, 2009

Homework: (Total 100 points) Do the following exercises.

For a question that requires the specification of a finite automaton, in the absence of further instructions, your solution may use any one of the notations in the textbook (five-tuple, transition diagram, or transition table).

Problem 1. (10 points) Provide DFAs for the following languages over the alphabet $\Sigma = \{0, 1\}$.
(a). All strings that contain *at least two* instances of the substring 01.
(b). All strings that *do not end* with 111.

Problem 2. (20 points)
(a). [5 points] Let $L \subset \{0, 1\}^*$ be the language of all strings such that **there are** two 0's separated by a number of positions that is a *non-zero* multiple of 5. Each position between the two 0's contains an arbitrary symbol (0 or 1) from the alphabet. For example, 1001110 is not in L, but $1\underline{0}11111\underline{0}$ and $\underline{0}10101\underline{0}1$ are in L. Construct an NFA for this language.
(b). [15 points] You should be glad that I did not ask you to construct a DFA for this language. To truly appreciate this, prove that any DFA for this language must have at least 2^5 states.
(*Food for thought:* Why does your argument fail for NFAs? How does the lower bound on the DFA size compare with your NFA's size? What are the implications for the relative power of NFAs and DFAs?)

Problem 3. (20 points) Consider the NFA $N = (Q, \Sigma, \delta, q_0, F)$ with $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{0, 1\}$, $F = \{q_2\}$, and the transition function as defined in the following table.

δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_1, q_2\}$	$\emptyset$
q_2	$\emptyset$	$\{q_2\}$

(a). Describe the language defined by this NFA.
(b). Convert this NFA into a DFA using the subset construction described in class. Your solution should consist of a transition diagram with *only* the essential states.

Problem 4. (15 points) Consider an NFA $N_1 = (Q, \Sigma, \delta, p, F_1)$ with language $L_1 = L(N_1)$. Define a new NFA $N_2 = (Q, \Sigma, \delta, p, F_2)$ with the set of final states $F_2 = Q - F_1$. (That is, going from N_1 to N_2 the final states become non-final, and vice-versa.)

Prove or disprove: *The language* $L_2 = L(N_2)$ *is the* **complement** *of the language* L_1*, i.e.,* $L_2 = \Sigma^* - L_1$.

(**Note:** If the statement is true, you must provide a formal proof that applies to all NFAs N_1. However, if the statement is false, then all you need to do is describe a specific NFA N_1 and show that the statement is incorrect when applied to this N_1.)

Problem 5. (15 points) Consider the ϵ-NFA defined in the following transition table.

	ϵ	a	b	c
$\rightarrow p$	$\{q, r\}$	$\{q\}$	$\{r\}$	$\emptyset$
q	$\{p, q\}$	$\emptyset$	$\{p\}$	$\{p, r\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

a). [5 points] Compute the ϵ-closure of each state.
b). [10 points] Convert this ϵ-NFA into an DFA using the construction described in class. You must provide the transition table of the resulting DFA.

Problem 6. (20 points) Provide regular expressions for the following languages over the alphabet $\Sigma = \{0, 1\}$. Provide a brief explanation as to why your regular expressions generate the given languages.
a). [10 points] The set of all strings not containing 111 as a substring.
b). [10 points] The set of all strings in which all pairs of adjacent 0's appear before all pairs of adjacent 1's.