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10: Normal Distributions

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[Lecture Discussion on Ed]

Normal Random Variables

Normal Random Variable

def A **Normal** random variable X is defined as follows:

expectation, center of gravity

variance

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

Support: $(-\infty, \infty)$

PDF	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
Expectation	$E[X] = \mu$
Variance	$\text{Var}(X) = \sigma^2$

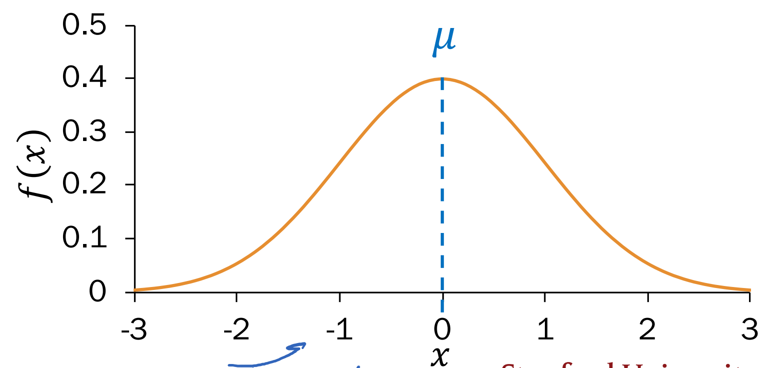
Other names: **Gaussian** random variable

I call it this much more often than most people

mean

variance

$$X \sim \mathcal{N}(\mu, \sigma^2)$$



units are standard deviations from μ

Carl Friedrich Gauss

Carl Friedrich Gauss (1777-1855) was a remarkably influential German mathematician.

Johann Carl Friedrich Gauss (/ɡaʊs/; German: *Gauß* [ɡaʊs] listen); Latin: *Carolus Fridericus Gauss*; 30 April 1777 – 23 February 1855) was a German mathematician and physicist who made significant contributions to many fields, including algebra, analysis, astronomy, differential geometry, electrostatics, geodesy, geophysics, magnetic fields, matrix theory, mechanics, number theory, optics and statistics.

just wow!

Sometimes referred to as the *Princeps mathematicorum*[1] (Latin for "the foremost of mathematicians") and "the greatest mathematician since antiquity", Gauss had an exceptional influence in many fields of mathematics and science, and is ranked among history's most influential mathematicians.[2]

Did **not** invent Normal distribution but rather popularized it.

Why the Normal?

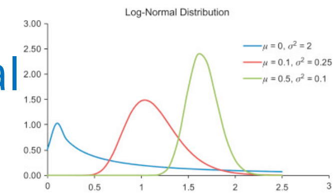
- Common for natural phenomena: height, weight, etc.
- Most noise in the world is Normal
- Often results from the sum of many random variables
- Sample means are distributed normally

That's what they want you to believe…

Why the Normal?

- Common for natural phenomena: height, weight, etc.

Actually log-normal



- Most noise in the world is Normal

Just an assumption } though it's a reasonably good one

- Often results from the sum of many random variables

Only if equally weighted

we will study this!

- Sample means are distributed normally

(okay this one is true, we'll see this in 3 weeks)

Okay, so why the Normal?

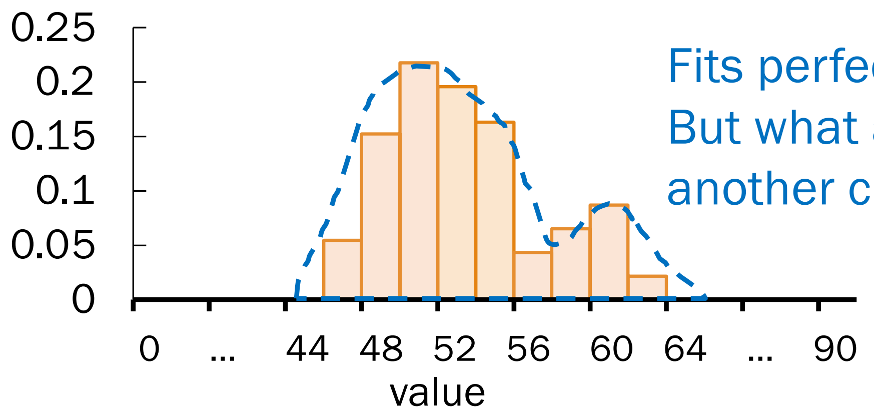
Part of CS109 learning goals:

- Translate a problem statement into a random variable

In other words: model real life situations with probability distributions

How do you model student heights?

- Suppose you have data from one classroom.



Fits perfectly!
But what about in another classroom?

Okay, so why the Normal?

maximizes entropy?
it means the probability distribution is as generic a distribution as one can choose and still do a good job modelling some problem.

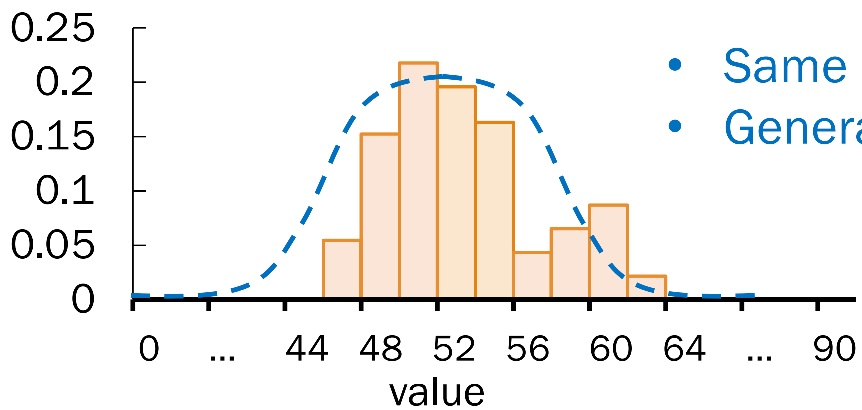
Part of CS109 learning goals:

- Translate a problem statement into a random variable

In other words: model real life situations with probability distributions

How do you model student heights?

- Suppose you have data from one classroom.



- Same mean/var
- Generalizes well

Occam's Razor:

"Non sunt multiplicanda entia sine necessitate."

Entities should not be multiplied without necessity.

A Gaussian maximizes entropy for a given mean and variance.

Why the Normal?

- Common for natural phenomena: height, weight, etc.
- Most noise in the world is Normal
- Often results from
random v
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Actually log-norr

mption

Only if equally weighted

(okay this one is true, we'll see this in 3 weeks)

because it's well understood

Stay critical of how to model real-world phenomena.

Anatomy of a beautiful equation

Let $X \sim \mathcal{N}(\mu, \sigma^2)$.

The PDF of X is defined as:

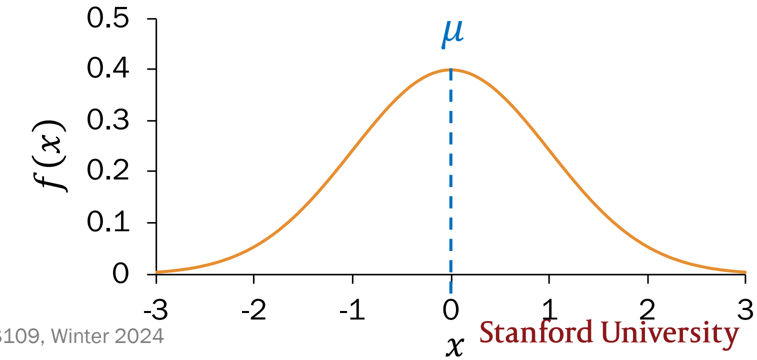
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

symmetric around μ

variance σ^2 manages spread

exponential tail

normalizing constant



Normal Random Variable

mean variance

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

Match PDF to distribution:

1. $\mathcal{N}(0, 1)$

A. of the three Gaussians centered at $x=0$, this one has the "middle" spread

2. $\mathcal{N}(-2, 0.5)$

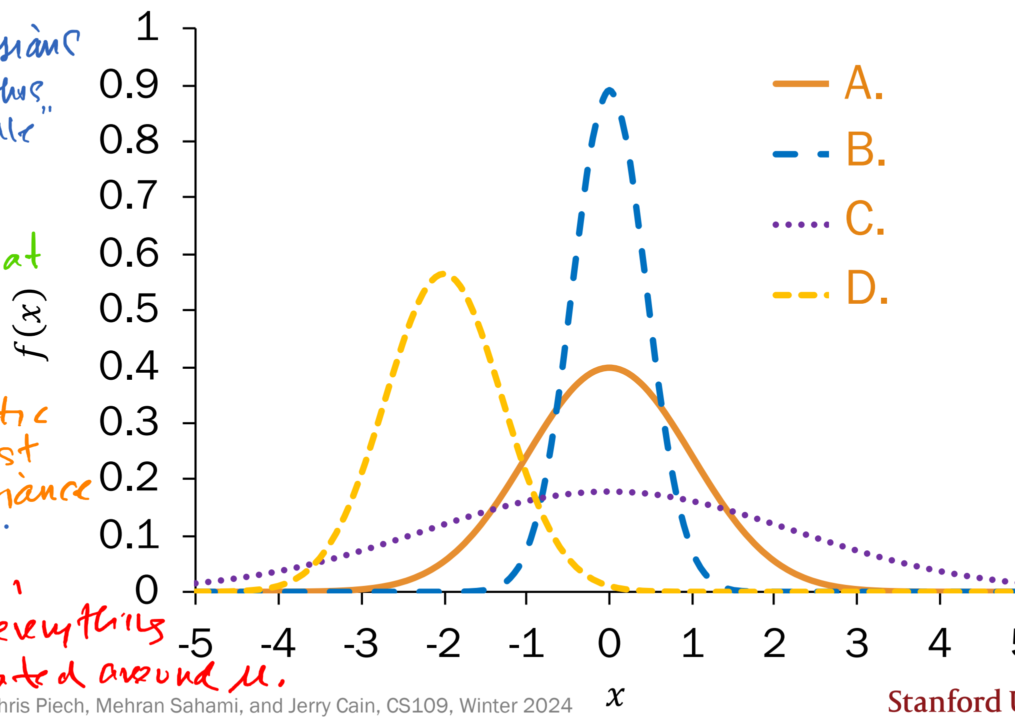
D this is the only one not centered at $x=0$

3. $\mathcal{N}(0, 5)$

C centered at $x=0$, has the most dramatic spread aka largest variance

4. $\mathcal{N}(0, 0.2)$

B centered at $x=0$, tiny variance means everything is super concentrated around μ.



Getting to class

You spend some minutes, X, traveling between classes.

- Average time spent: $\mu = 4$ minutes
- Variance of time spent: $\sigma^2 = 2$ minutes^2

Suppose X is normally distributed. What is the probability you spend ≥ 6 minutes traveling?

$$X \sim \mathcal{N}(\mu = 4, \sigma^2 = 2)$$

$$P(X \geq 6) = \int_6^\infty f(x)dx = \int_6^\infty \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-\overset{\mu}{4})^2}{\underset{2\sigma^2}{4}}} dx$$

(tell Jerry if you solve this analytically and we'll be famous together)

Love and Anger in the Same Formula

Computing probabilities with Normal RVs

For a Normal RV $X \sim \mathcal{N}(\mu, \sigma^2)$, its CDF has no closed form.

$$P(X \le x) = F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy$$

⚠ Cannot be solved analytically

However, we can solve for probabilities numerically using a function Φ:

argument is the number of standard deviations away from the mean.

$$F(x) = \Phi\left(\overbrace{\frac{x-\mu}{\sigma}}\right)$$

To get here, we'll first need to know some properties of Normal RVs.

CDF of $X \sim \mathcal{N}(\mu, \sigma^2)$

A function that has been solved **numerically**

Normal RV: Properties

Properties of Normal RVs

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with CDF $P(X \le x) = F(x)$.

1. Linear transformations of Normal RVs are also Normal RVs.

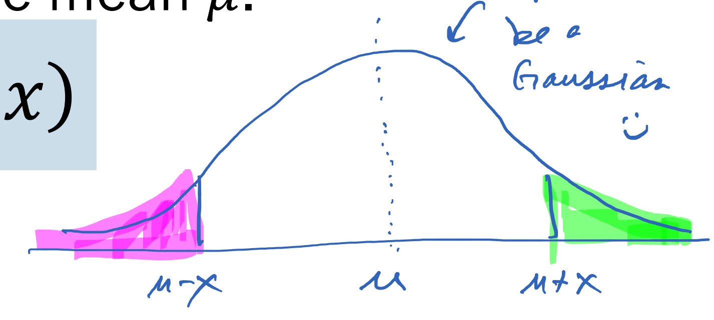
$$\text{If } Y = aX + b, \text{ then } Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2).$$

2. The PDF of a Normal RV is symmetric about the mean μ.

$$F(\mu - x) = 1 - F(\mu + x)$$

this says the area in purple matches the area in green

supposed to be a Gaussian :)



1. Linear transformations of Normal RVs

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with CDF $P(X \leq x) = F(x)$.

Linear transformations of X are also Normal.

If $Y = aX + b$, then $Y \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

Proof:

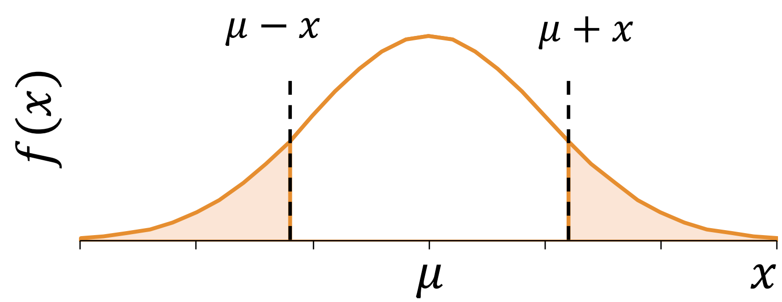
- $E[Y] = E[aX + b] = aE[X] + b = a\mu + b$ — Linearity of Expectation
- $\text{Var}(Y) = \text{Var}(aX + b) = a^2\text{Var}(X) = a^2\sigma^2$ — $\text{Var}(aX + b) = a^2\text{Var}(X)$
- Y is also Normal — Proof in Ross, 10th ed (Section 5.4)

2. Symmetry of Normal RVs

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with CDF $P(X \leq x) = F(x)$.

The PDF of a Normal RV is symmetric about the mean μ.

$$F(\mu - x) = 1 - F(\mu + x)$$



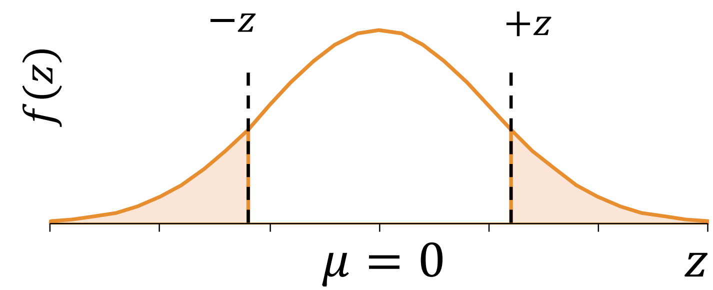
Using symmetry of the Normal RV

$F(\mu - x) = 1 - F(\mu + x)$

Let $Z \sim \mathcal{N}(0,1)$ with CDF $P(Z \leq z) = F(z)$.

Suppose we only knew numeric values for $F(z)$ and $F(y)$, for some $y, z \geq 0$.

How do we compute the following probabilities?



1. $P(Z \leq z)$ $= F(z)$
2. $P(Z < z)$
3. $P(Z \geq z)$
4. $P(Z \leq -z)$
5. $P(Z \geq -z)$
6. $P(y < Z < z)$

A. $F(z)$

B. $1 - F(z)$

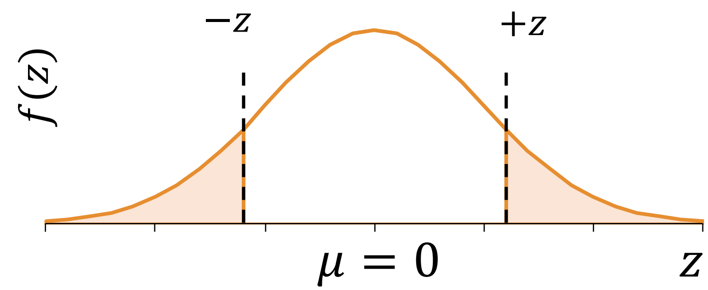
C. $F(z) - F(y)$

Using symmetry of the Normal RV

$F(\mu - x) = 1 - F(\mu + x)$

Let $Z \sim \mathcal{N}(0,1)$ with CDF $P(Z \leq z) = F(z)$.

Suppose we only knew numeric values for $F(z)$ and $F(y)$, for some $y, z \geq 0$.



How do we compute the following probabilities?

1. $P(Z \leq z) \quad = F(z)$
2. $P(Z < z) \quad = F(z)$
3. $P(Z \geq z) \quad = 1 - F(z)$
4. $P(Z \leq -z) \quad = 1 - F(z)$
5. $P(Z \geq -z) \quad = F(z)$
6. $P(y < Z < z) \quad = F(z) - F(y)$

A. $F(z)$

B. $1 - F(z)$

C. $F(z) - F(y)$

Symmetry is particularly useful when computing probabilities of zero-mean Normal RVs.

Normal RV: Computing probability

Computing probabilities with Normal RVs

Let $X \sim \mathcal{N}(\mu, \sigma^2)$.

To compute the CDF, $P(X \leq x) = F(x)$:

- We cannot analytically solve the integral, as it has no closed form.
- … but we ***can*** solve numerically using a function Φ:

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

CDF of the
Standard Normal, Z

Standard Normal RV, Z

The **Standard Normal** random variable Z is defined as follows:

$$Z \sim \mathcal{N}(0, 1)$$

Expectation $E[Z] = \mu = 0$

Variance $\text{Var}(Z) = \sigma^2 = 1$

Note: not a new distribution; just a special case of the Normal

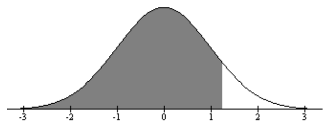
Other names: **Unit Normal** ← Jerry says this one more often.

CDF of Z defined as: $P(Z \leq z) = \Phi(z)$

Φ has been numerically computed

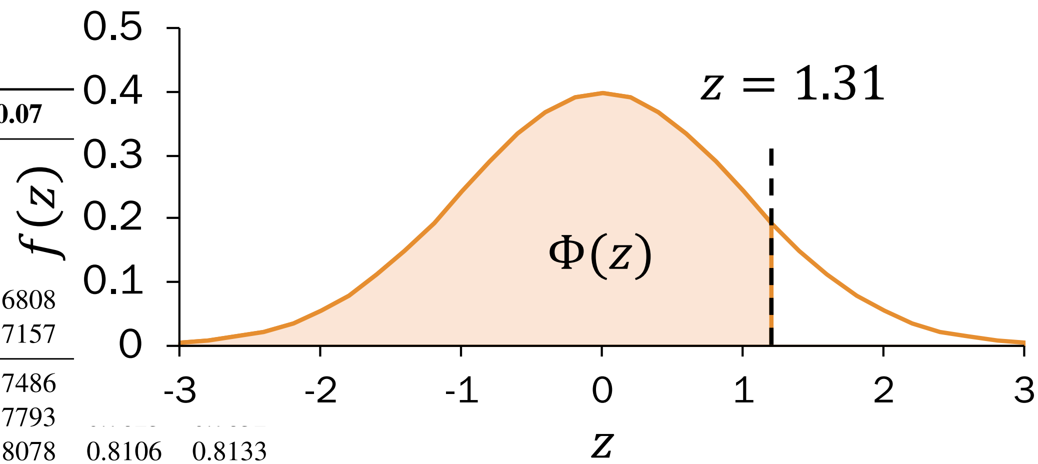
Standard Normal Table

An entry in the table is the area under the curve to the left of z, $P(Z \le z) = \Phi(z)$.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07		
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0		
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0		
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0		
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0		
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808		
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157		
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486		
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7793		
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	$\boxed{0.9049}$	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

$$P(Z \le 1.31) = \Phi(1.31)$$



Standard Normal Table only has probabilities $\Phi(z)$ for $z \ge 0$.

History fact: Standard Normal Table

TABLES

SERVANT

AU CALCUL DES RÉFRACTIONS

APPROCHANTES DE L'HORIZON.

TABLE PREMIÈRE.

Intégrales de $e^{-tt}\,dt$, depuis une valeur quelconque de t jusqu'à t infinie.

t	*Intégrale.*	*Diff. prem.*	*Diff. II.*	*Diff. III.*
0,00	0,88622692	999968	201	199
0,01	0,87622724	999767	400	199
0,02	0,86622057	999367	599	200
0,03	$\boxed{0,85623590}$	998768	799	199
0,04	0,84624822	997969	998	197
0,05	0,83626853	996971	1195	199
0,06	0,82629882	995776	1394	196

The first Standard Normal Table was computed by Christian Kramp, French astronomer (1760–1826), in *Analyse des Réfractions Astronomiques et Terrestres*, 1799

Used a Taylor series expansion to the third power

integral from x = 0.03 to infinity of e^{-x^2}

Extended Keyboard   Upload

Definite integral:

$$\int_{0.03}^{\infty} e^{-x^2}\,dx = 0.856236$$

Probabilities for a general Normal RV

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. To compute the CDF $P(X \leq x) = F(x)$, we use Φ, the CDF for the Standard Normal $Z \sim \mathcal{N}(0, 1)$:

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

Proof:

define new variable Z to be $\frac{X-\mu}{\sigma}$

$$F(x) = P(X \leq x)$$ Definition of CDF

$$= P(X - \mu \leq x - \mu) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right)$$ Algebra + $\sigma > 0$

$$= P\left(Z \leq \frac{x-\mu}{\sigma}\right)$$

$$= \Phi\left(\frac{x-\mu}{\sigma}\right)$$

- $\frac{X-\mu}{\sigma} = \frac{1}{\sigma}X - \frac{\mu}{\sigma}$ is a linear transform of X.
- This is distributed as $\mathcal{N}\left(\frac{1}{\sigma}\mu - \frac{\mu}{\sigma}, \frac{1}{\sigma^2}\sigma^2\right) = \mathcal{N}(0,1)$
- In other words, $\frac{X-\mu}{\sigma} = Z \sim \mathcal{N}(0,1)$ with CDF Φ.

Probabilities for a general Normal RV

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. To compute the CDF $P(X \leq x) = F(x)$, we use Φ, the CDF for the Standard Normal $Z \sim \mathcal{N}(0, 1)$:

$$F(x) = \Phi\left(\frac{x - \mu}{\sigma}\right)$$

Proof:

$$F(x) = P(X \leq x) \quad \text{Definition of CDF}$$

$$= P(X - \mu \leq x - \mu) = P\left(\frac{X - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}\right) \quad \text{Algebra} + \sigma > 0$$

$$= P\left(Z \leq \frac{x - \mu}{\sigma}\right)$$

$$= \Phi\left(\frac{x - \mu}{\sigma}\right)$$

- $\frac{X-\mu}{\sigma} = \frac{1}{\sigma}X - \frac{\mu}{\sigma}$ is a linear transform of X.

1. Compute $z = (x - \mu)/\sigma$.
2. Look up $\Phi(z)$ in Standard Normal table.

Campus bikes

You spend some minutes, X, traveling between classes.

- Average time spent: $\mu = 4$ minutes
- Variance of time spent: $\sigma^2 = 2$ minutes^2

Suppose X is normally distributed. What is the probability you spend ≥ 6 minutes traveling?

$$X \sim \mathcal{N}(\mu = 4, \sigma^2 = 2)$$

✕ $P(X \geq 6) = \int_6^\infty f(x)dx$ (no analytic solution)

1. Compute $z = \frac{(x-\mu)}{\sigma}$

$$P(X \geq 6) = 1 - F_x(6)$$
$$= 1 - \Phi\left(\frac{6-4}{\sqrt{2}}\right)$$
$$\approx 1 - \Phi(1.41)$$

2. Look up $\Phi(z)$ in table

$$1 - \Phi(1.41)$$
$$\approx 1 - 0.9207$$
$$= 0.0793$$

Is there an easier way? (yes)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$. What is $P(X \leq x) = F(x)$?

- Use Python

```
from scipy import stats
X = stats.norm(mu, std)
X.cdf(x)
```

SciPy reference:
https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html

’m not sure why Python decided to parameterize `stats.norm` around the standard deviation instead of the variance, but it did. ☺

Exercises

Get your Gaussian On

Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$. Std deviation $\sigma = 4$.

1. $P(X > 0)$ $= 1 - P(X < 0)$

$= 1 - F(0)$

$= 1 - \Phi\left(\frac{0-3}{4}\right)$

$= 1 - \Phi\left(\frac{-3}{4}\right)$

$= 1 - \left(1 - \Phi\left(\frac{3}{4}\right)\right)$

$= \Phi(3/4) = \underbrace{\boxed{.7734}}_{}$

look up in that table (or use Python's scipy library to get it) ☺

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
- Symmetry of the PDF of Normal RV implies $\Phi(-z) = 1 - \Phi(z)$

Get your Gaussian On

Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$.
Note standard deviation $\sigma = 4$.

How would you write each of the below probabilities as a function of the standard normal CDF, Φ?

1. $P(X > 0)$
2. $P(2 < X < 5)$
3. $P(|X - 3| > 6)$

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
- Symmetry of the PDF of Normal RV implies $\Phi(-z) = 1 - \Phi(z)$

Get your Gaussian On

Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$. Std deviation $\sigma = 4$.

1. $P(X > 0)$
2. $P(2 < X < 5)$ $= F(5) - F(2)$

$$= \phi\left(\frac{5-3}{4}\right) - \phi\left(\frac{2-3}{4}\right)$$

$$= \phi\left(\frac{1}{2}\right) - \phi\left(\frac{-1}{4}\right)$$

$$= \phi\left(\frac{1}{2}\right) - \left(1 - \phi\left(\frac{1}{4}\right)\right)$$

$$= \phi\left(\frac{1}{2}\right) + \phi\left(\frac{1}{4}\right) - 1$$

$$= \boxed{0.2902}$$

look these up in table

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
- Symmetry of the PDF of Normal RV implies $\Phi(-z) = 1 - \Phi(z)$

Get your Gaussian On

Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$. Std deviation $\sigma = 4$.

1. $P(X > 0)$
2. $P(2 < X < 5)$
3. $P(|X - 3| > 6)$

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
- Symmetry of the PDF of Normal RV implies $\Phi(-x) = 1 - \Phi(x)$

Compute $z = \frac{(x-\mu)}{\sigma}$

Look up $\Phi(\mathrm{z})$ in table

$$P(X < -3) + P(X > 9)$$

$$= F(-3) + \big(1 - F(9)\big)$$

$$= \Phi\left(\frac{-3-3}{4}\right) + \left(1 - \Phi\left(\frac{9-3}{4}\right)\right)$$

Get your Gaussian On

Let $X \sim \mathcal{N}(\mu = 3, \sigma^2 = 16)$. Std deviation $\sigma = 4$.

1. $P(X > 0)$
2. $P(2 < X < 5)$
3. $P(|X - 3| > 6)$

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
- Symmetry of the PDF of Normal RV implies $\Phi(-x) = 1 - \Phi(x)$

Compute $z = \frac{(x-\mu)}{\sigma}$

$$P(X < -3) + P(X > 9)$$

$$= F(-3) + \big(1 - F(9)\big)$$

$$= \Phi\left(\frac{-3-3}{4}\right) + \left(1 - \Phi\left(\frac{9-3}{4}\right)\right)$$

Look up $\Phi(z)$ in table

$$= \Phi\left(-\frac{3}{2}\right) + \left(1 - \Phi\left(\frac{3}{2}\right)\right)$$

$$= 2\left(1 - \Phi\left(\frac{3}{2}\right)\right)$$

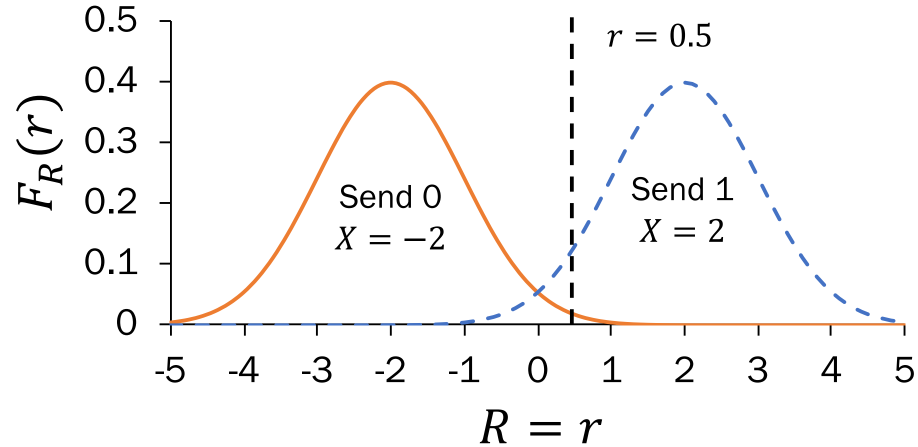
$$\approx 0.1337$$

yay!

Noisy Wires

Send a voltage of 2 V or -2 V on wire (to denote 1 and 0, respectively).

- $X =$ voltage sent (2 or -2)
- $Y =$ noise, $Y \sim \mathcal{N}(0, 1)$
- $R = X + Y$ voltage received.



Decode: 1 if $R \geq 0.5$
0 otherwise.

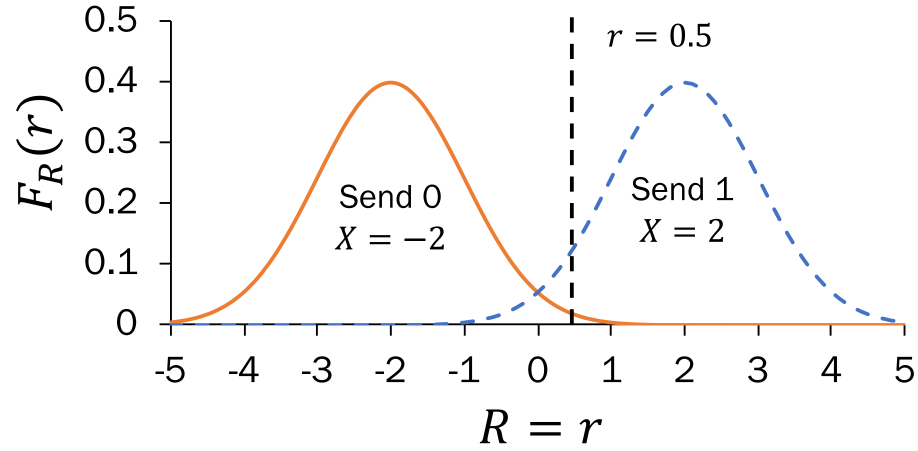
1. What is P(decoding error | original bit is 1)?
   i.e., we sent 1, but we decoded as 0?
2. What is P(decoding error | original bit is 0)?

These probabilities are unequal. Why might this be useful?

Noisy Wires

Send a voltage of 2 V or -2 V on wire (to denote 1 and 0, respectively).

- X = voltage sent (2 or -2)
- Y = noise, $Y \sim \mathcal{N}(0, 1)$
- $R = X + Y$ voltage received.



Decode: $\begin{cases} 1 & \text{if } R \geq 0.5 \\ 0 & \text{otherwise.} \end{cases}$

1. What is P(decoding error | original bit is 1)?
   i.e., we sent 1, but we decoded as 0?

$$P(R < 0.5 | X = 2) = P(2 + Y < 0.5) = P(Y < -1.5)$$

Y is Standard Normal

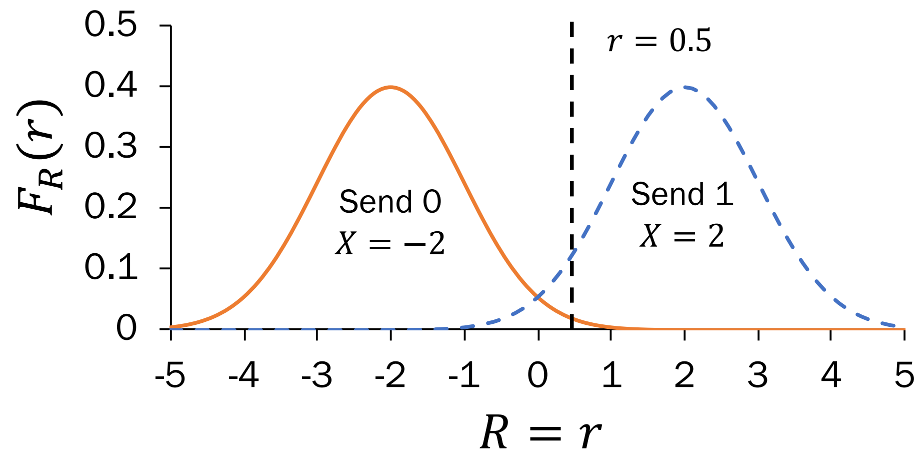
$$= \Phi(-1.5) = 1 - \Phi(1.5) \approx 0.0668$$

Noisy Wires

Send a voltage of 2 V or -2 V on wire (to denote 1 and 0, respectively).

- X = voltage sent (2 or -2)
- Y = noise, $Y \sim \mathcal{N}(0, 1)$
- $R = X + Y$ voltage received.

Decode: $\begin{cases} 1 & \text{if } R \geq 0.5 \\ 0 & \text{otherwise.} \end{cases}$



1. What is P(decoding error | original bit is 1)? i.e., we sent 1, but we decoded as 0?

   0.0668

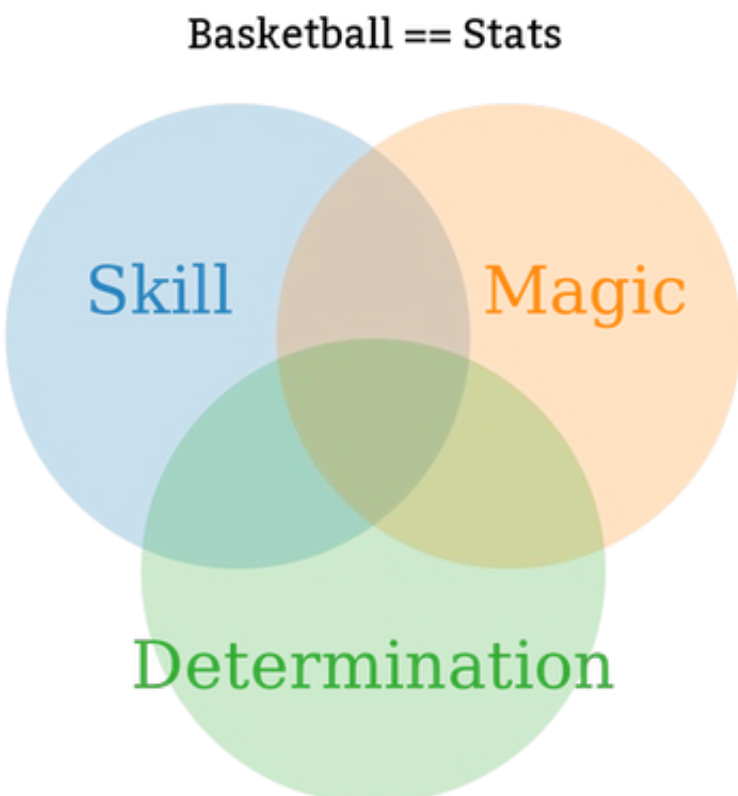
2. What is P(decoding error | original bit is 0)?

$$P(R \geq 0.5 \mid X = -2) = P(-2 + Y \geq 0.5) = \overbrace{P(Y \geq 2.5)}^{1-\phi(2.5)} \approx 0.0062$$

Asymmetric decoding probability: We would like to avoid mistaking a 0 for 1. Errors the other way are tolerable.

Sampling with the Normal RV

ELO ratings



What is the probability that the Warriors win?
More generally: How can you model zero-sum games?

ELO ratings

Arpad Elo

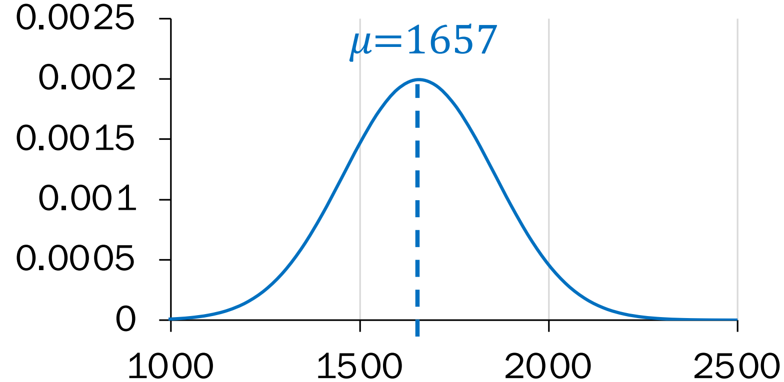
Each team has an ELO score S, calculated based on its past performance.

- Each game, a team has ability $A \sim \mathcal{N}(S, 200^2)$.
- The team with the higher sampled ability wins.

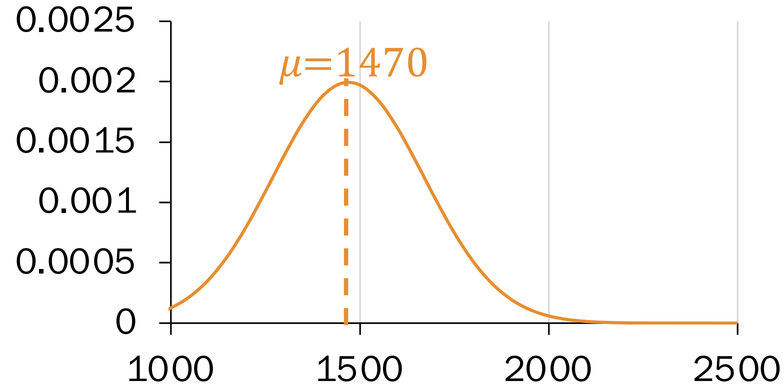
What is the probability that Warriors win this game?

Want: $P(\text{Warriors win}) = P(A_W > A_O)$

Warriors $A_W \sim \mathcal{N}(S = 1657, 200^2)$



Opponents $A_O \sim \mathcal{N}(S = 1470, 200^2)$



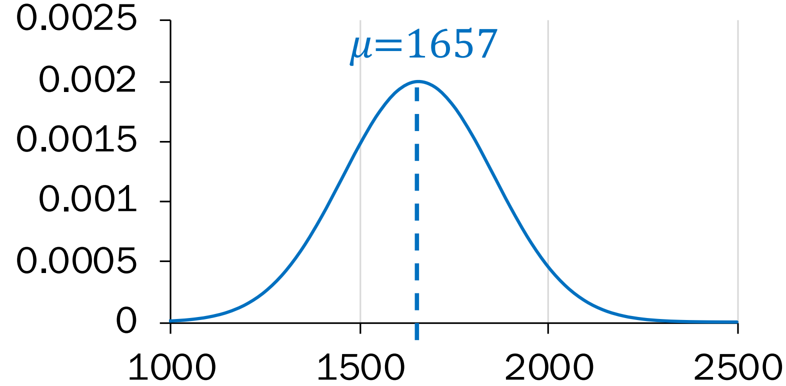
ELO ratings

Want: $P(\text{Warriors win}) = P(A_W > A_O)$

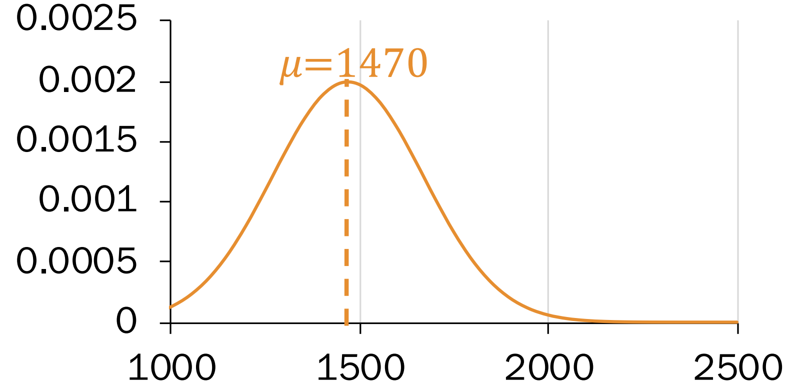
```python
from scipy import stats
WARRIORS_ELO = 1657
OPPONENT_ELO = 1470
STDEV = 200
NTRIALS = 10000

nSuccess = 0
for i in range(NTRIALS):
  w = stats.norm.rvs(WARRIORS_ELO, STDEV)
  o = stats.norm.rvs(OPPONENT_ELO, STDEV)
  if w > o: nSuccess += 1

print("Warriors sampled win fraction",
      float(nSuccess) / NTRIALS)
```

≈ 0.7488, calculated by sampling

Warriors $A_W \sim \mathcal{N}(S = 1657, 200^2)$



Opponents $A_O \sim \mathcal{N}(S = 1470, 200^2)$



Is there a better way?

$$P(A_W > A_O)$$

actual depiction of someone understanding joint continuous random variables

- This is a probability of an event involving **two continuous random variables!**
- We'll solve this problem analytically in less than two weeks' time.

Big goal for next lecture: Events involving two **discrete** random variables.

Stay tuned!

ChatGPT Takeaway

What natural phenomena are often modeled as Gaussian even though they're better modeled by other distributions?

There are some natural phenomena that are often modeled as Gaussian distributions even though they are better modeled by other distributions. Here are a few examples:

I've not seen log-Normal used here in practice, though I have seen Pareto used!

1. Income distribution: While it is common to assume that income follows a Gaussian distribution, it is actually better modeled using other distributions such as the log-normal or Pareto distributions.
2. Stock prices: Although stock prices are often assumed to follow a Gaussian distribution, this assumption does not capture the large and sudden changes in prices that can occur. A more appropriate model is the fat-tailed distribution, such as the Cauchy distribution.
3. Extreme weather events: Extreme weather events such as hurricanes or tornadoes do not follow a Gaussian distribution. Rather, they are better modeled using a power-law distribution, which captures the fact that these events are rare but can have very large magnitudes.
4. Reaction times: The distribution of reaction times in tasks such as decision-making or perceptual tasks is often assumed to be Gaussian. However, recent research has shown that these distributions are better described by ex-Gaussian or Lévy distributions.