

CS103X: Discrete Structures

Homework Assignment 7

Due March 14, 2008

Exercise 1 (20 points). Let G be a graph that has no induced subgraphs that are P_4 or C_3 .

- (a) Prove that G is bipartite.
- (b) Assume in addition that G is connected. Prove that G is a complete bipartite graph.

Exercise 2 (15 points). Given a bipartite graph G , prove that its two classes are unique (up to interchanging their order) if and only if G is connected.

Exercise 3 (15 points). For any $k \in \mathbb{N}^+$, prove that a k -regular bipartite graph has a perfect matching.

Exercise 4 (15 points). Given a tree G that contains a vertex of degree k , prove that G has at least k leaves.

Exercise 5 (15 points). Prove that $G = (V, E)$ is a tree if and only if $|V| = |E| + 1$ and G has no cycles.

Exercise 6 (20 points). Let G be a simple graph with n vertices and k connected components.

- (a) What is the minimum possible number of edges of G ?
- (b) What is the maximum possible number of edges of G ?