# CS 103X: Discrete Structures
# Homework Assignment 5

Due February 17, 2006

**Exercise 1** (25 points)**.** Formulate each of the below as a *single* statement (proposition or predicate), using only mathematical and logical notation that has been defined in class. For example, the use of logical quantifiers and connectives, and arithmetic, number-theoretic, and set-theoretic operations is allowed, as is the use of sets like $\mathbb{Q}$, $\mathbb{R}$, etc., but not the use of English-language words.

(a) Every positive integer is either even or odd.

(b) $p$ is prime.

(c) There are infinitely many primes.

(d) If $a$ and $b$ are integers and $b \neq 0$, then there is a unique pair of integers $q$ and $r$, such that $a = qb + r$ and $0 \leq r < |b|$.

(e) If $a$ and $n$ are coprime then there exists exactly one $x \in \mathbb{Z}_n$ for which $ax \equiv_n b$, for any $b \in \mathbb{Z}$.

**Exercise 2** (25 points)**.** After completing the previous exercise, write the negation of each of your logical statements, such that the symbol $\neg$ does not appear in you statements. (That is, eliminate negated quantifiers and negated compounds as you have learned in class, and then replace statements such as $\neg(a|b)$ by statements like $a \nmid b$.) Read the negations out in natural language and check for yourself that you understand why these are the right negations for the statements in the previous exercise.

**Exercise 3** (25 points)**.** Which of the following are valid equivalences? Prove.

(a) $(P \to Q) \wedge (P \to R) \Leftrightarrow (P \to (Q \wedge R))$

(b) $(P \to R) \wedge (Q \to R) \Leftrightarrow ((P \wedge Q) \to R)$

(c) $(P \wedge Q) \vee R \Leftrightarrow (P \vee R) \wedge (Q \vee R)$

(d) $(P \vee Q) \wedge R \Leftrightarrow (P \wedge R) \vee (Q \wedge R)$

**Exercise 4** (25 points)**.** Define a new connective $\overline{\wedge}$ (read "nand") as follows:

| $P$ | $Q$ | $P \overline{\wedge} Q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | T |

Show that the propositions $\neg P$, $P \wedge Q$, and $P \vee Q$ can be expressed in terms of $\overline{\wedge}$ alone, with no other connectives. Conclude that $\overline{\wedge}$ is *universal*, meaning that a proposition that involves any of the connectives we have seen so far can be expressed using only $\overline{\wedge}$.