CS 103X: Discrete Structures
Homework Assignment 2

Due January 27, 2006

Exercise 1 (Reading Assignment). Read pages 35–36 in Lehman-Leighton.

Exercise 2. On induction and well-ordering:

(a) Prove the strong induction principle from the principle of induction.

(b) Prove the induction principle from the well-ordering principle.

(c) Prove the well-ordering principle from the induction principle.

Conclude that the principles of induction, strong induction, and well-ordering are equally powerful. That is, anything that can be derived from one, can also be derived from any of the others.

Exercise 3. More irrational numbers:

(a) Prove that $\sqrt{3}$ and $\sqrt{6}$ are irrational.

(b) Prove that $\sqrt{2} + \sqrt{3}$ and $\sqrt{2} + \sqrt{6}$ are irrational.

Exercise 4. For each of the following propositions, give a short proof or a counterexample. Assume $a, b, c, d, m \in \mathbb{Z}$.

(a) If $a|b$ and $c|d$ then $ac|bd$.

(b) If $a|b$ and $c|d$ then $(a + c)|(b + d)$.

(c) If $a|b$ then $a|bc$ for all c.

(d) If $m \neq 0$, then $a|b$ if and only if $ma|mb$.

Exercise 5. On Euclid's algorithm:

(a) Write the algorithm in pseudo-code.

(b) State a theorem that asserts the correctness of the algorithm and prove the theorem.

(c) Use the algorithm to calculate $\gcd(9524, 8692)$. Write out the complete sequence of derivations.

Exercise 6. EXTRA CREDIT:

As I mentioned, there is a lot of speculation concerning the Pythagoreans. Since legends abound, there is no harm in another one.

Once upon a time there was a Pythagorean sect with n members in the town of Kroton. They would all meet once a week and share their newest mathematical discoveries. The members upheld a strict code of honor that included a commitment to publicly taking one's own life at the next week's meeting if one ever realizes that a result that one previously shared with the group was erroneous. In reality, each member had committed mathematical errors in the past, which were noticed and discussed by all other members among themselves outside the weekly meetings. The one committing the error was never informed of it, as the Pythagoreans were peaceful folk who did not want to see their friend die. Thus each member of the sect had made an error at some past meeting, was unaware of it, but all the others knew of it, and also knew that they all knew because they discussed it among themselves.

After years of peaceful existence the sect was infiltrated by a Sybarite, who learned of this state of affairs through private conversations with each of the sectarians. One time at a weekly meeting, the Sybarite stood up and announced: "It has come to my knowledge that at least one of you had made a mathematical error in the past, and this is known to all the others." After which the infiltrator promptly departed.

Assuming that the Pythagoreans, true to their nature, behave with perfect rationality and integrity, describe and prove precisely what the consequences of this announcement were.