CS103A HO#44 11/3/08

Slides--Sequences, Induction

CS103A

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Logic and formal proofs

Proving mathematical theorems (number theory)

Crypto

Sequences and summations

Mathematical induction

Recursion

Combinatorics

Functions

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Sequences and Summations

A sequence is an ordered list, possibly infinite, of elements.

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a_k =

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$a_k = a_{k-1} + 2k - 1$ where $a_1 = 3$

---

k: 1 2 3 4 5 6 7 8

a_k: 7, 11, 15, 19, 23, 27, 31, 35 ...

Explicit formula $a_k =$

Recursive formula $a_1 = 7$

$a_k =$

---

k: 1 2 3 4 5 6 7 8

a_k: 7, 11, 15, 19, 23, 27, 31, 35 ...

Explicit formula $a_k = 4k + 3$

Recursive formula $a_1 = 7$

$a_k = a_{k-1} + 4$

---

k: 1 2 3 4 5 6 7 8 9 10

a_k: 0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682 ...

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$a_k =$

k:	1	2	3	4	5	6	7	8	9	10
a_k:	0,	2,	8,	26,	80,	242,	728,	2186,	6560,	19682 ...

Explicit formula $a_k = 3^{k-1} - 1$

Recursive formula $a_1 = 0$

$a_k = a_{k-1} + 2 \cdot 3^{k-2}$

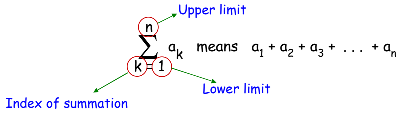
or $a_k = 3a_{k-1} + 2$

Sum and Product Notation

$\sum_{k=1}^{n} a_k$ means $a_1 + a_2 + a_3 + \ldots + a_n$

$\prod_{k=1}^{n} a_k$ means $a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$

Sum and Product Notation



Upper limit

$\sum_{k=1}^{n} a_k$ means $a_1 + a_2 + a_3 + \ldots + a_n$

Lower limit

Index of summation

$\prod_{k=1}^{n} a_k$ means $a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$

Linearity Property of Sums

$$\sum_{k=1}^{n} (c \cdot a_k + b_k) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

Interesting cases: $c = 1$
$b_k = 0$ for $k = 1,\ldots,n$

What is the value of $\sum_{i=1}^{4} \sum_{j=1}^{3} i\,j$?

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Writing out the j sum: $\sum_{i=1}^{4} (i + 2i + 3i) = \sum_{i=1}^{4} 6i = 6\sum_{i=1}^{4} i$

$= 6 \cdot 10 = 60$

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Factoring out i first: $\sum_{i=1}^{4} \left(i \sum_{j=1}^{3} j \right) = \sum_{i=1}^{4} 6i = 60$

Arithmetic Progressions

An arithmetic progression is a sequence of the form

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)\,d, \ldots$

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An arithmetic progression is a sequence of the form

$$\underbrace{a}_{\text{Initial term}}, a + \underbrace{d}_{\text{Common difference}}, a + 2d, a + 3d, \ldots, \underbrace{a + (n - 1)\,d}_{n^{th}\text{ term if we number from 1}}, \ldots$$

Arithmetic Progressions

An arithmetic progression is a sequence of the form

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)\,d, \ldots$

Explicit formula: $t_k = a + (k - 1)\,d$

Recursive formula: $t_1 = a$

$t_k = t_{k-1} + d$

Arithmetic Progressions

k:	1	2	3	4	5	6	7	8
a_k:	7,	11,	15,	19,	23,	27,	31,	35 ...

Explicit formula $a_k = 7 + (k - 1)\,4 = 4k + 3$

Recursive formula $a_k = a_{k-1} + 4$

Arithmetic Progressions

An arithmetic progression is a sequence of the form

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)\,d, \ldots$

If $a = 1$ and $d = 1$, the sequence is $1, 2, 3, 4, \ldots$

The sum of an initial segment of an arithmetic progression is called an arithmetic series.

For $1, 2, 3, \ldots$ $\sum_{k=1}^{n} t_k = \sum_{k=1}^{n} k = \frac{n\,(n + 1)}{2}$

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In general $\quad \sum_{k=1}^{n} t_k = \frac{n (t_1 + t_n)}{2} = \frac{n (2a + (n - 1) d)}{2}$

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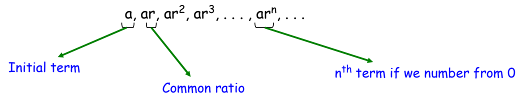

$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$

Initial term

Common ratio

n^{th} term if we number from 0

Geometric Progressions

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$a, ar, ar^2, ar^3, \ldots, ar^n, \ldots$

$$\sum_{i=0}^{n} ar^i = \begin{cases} \frac{a (r^{n+1} - 1)}{r - 1} & \text{if } r \neq 1 \\ a (n + 1) & \text{if } r = 1 \end{cases}$$

Geometric Progressions

Let's show that

$$S = \sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r} \quad \text{if } r \neq 1$$

$S = r^0 + r^1 + r^2 + \ldots + r^{n-1} + r^n$ (1) Equivalent to $\sum$ notation

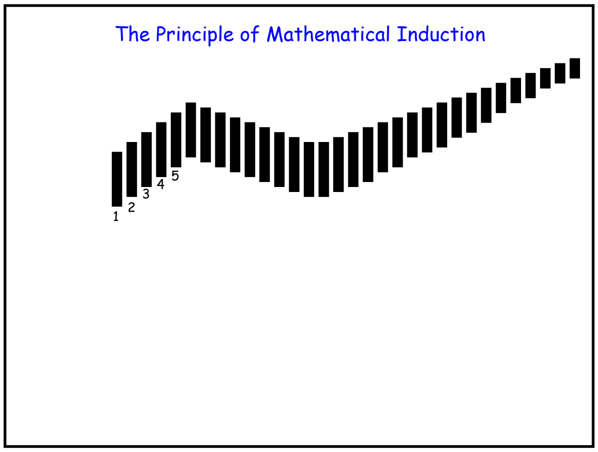
$rS = r^1 + r^2 + r^3 + \ldots r^n + r^{n+1}$ (2) Multiply by r

$S - rS = r^0 + r^1 + r^2 + \ldots + r^{n-1} + r^n - r^1 - r^2 - r^3 - \ldots - r^n - r^{n+1}$ (1) - (2)

$S (1 - r) = r^0 - r^{n+1}$ Factor LHS, cancel terms on RHS

$S = \frac{1 - r^{n+1}}{1 - r}$ $r^0 = 1$, and $r \neq 1$

The Principle of Mathematical Induction



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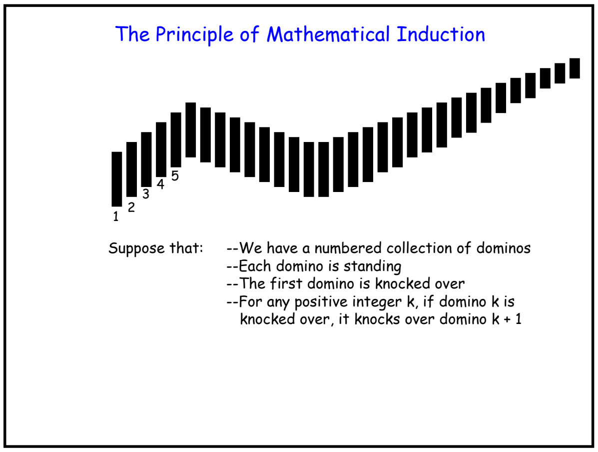
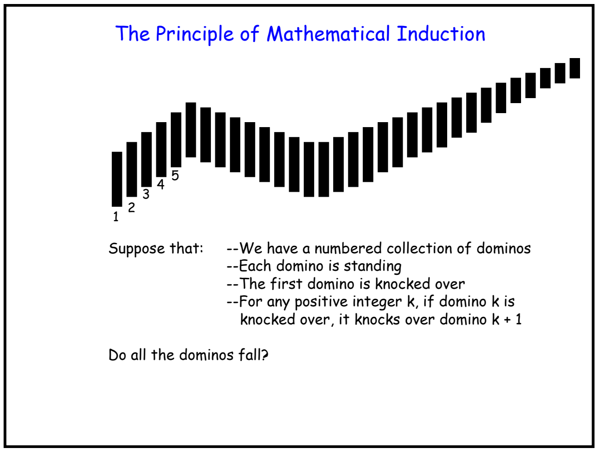


The Principle of Mathematical Induction

Suppose that: --We have a numbered collection of dominos
--Each domino is standing
--The first domino is knocked over
--For any positive integer k, if domino k is knocked over, it knocks over domino k + 1



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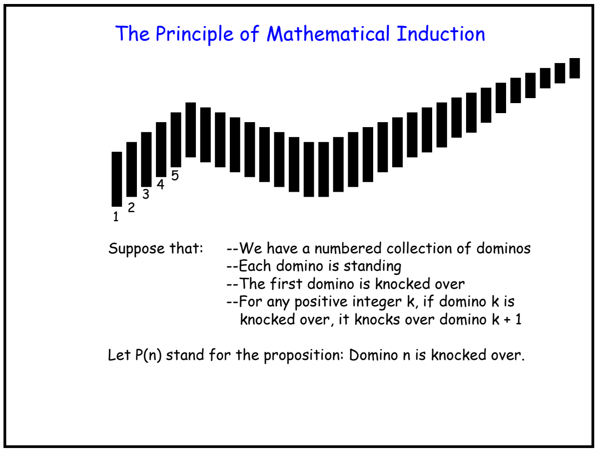
Do all the dominos fall?



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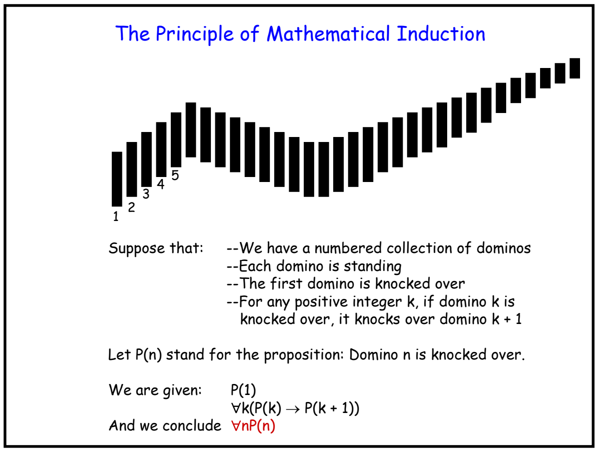
Let P(n) stand for the proposition: Domino n is knocked over.



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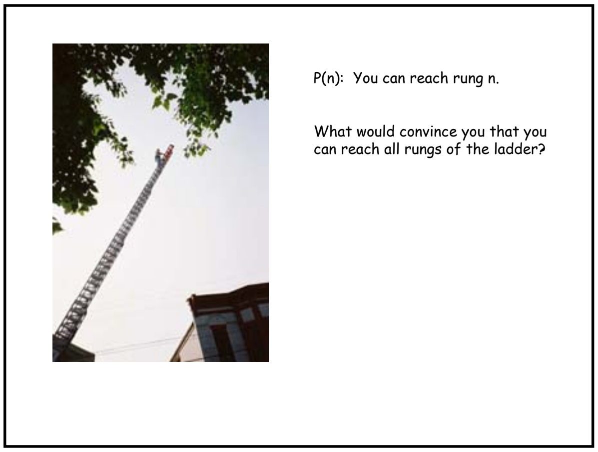
We are given: $P(1)$
$\forall k(P(k) \rightarrow P(k + 1))$
And we conclude $\forall n P(n)$



P(n): You can reach rung n.

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$\forall k(P(k) \rightarrow P(k+1))$: If you can reach rung k, you can reach rung k + 1.

If we can prove those two statements, then we can conclude: $\forall n P(n)$

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A proof by mathematical induction that a proposition P(n) is true for every positive integer n consists of two steps:

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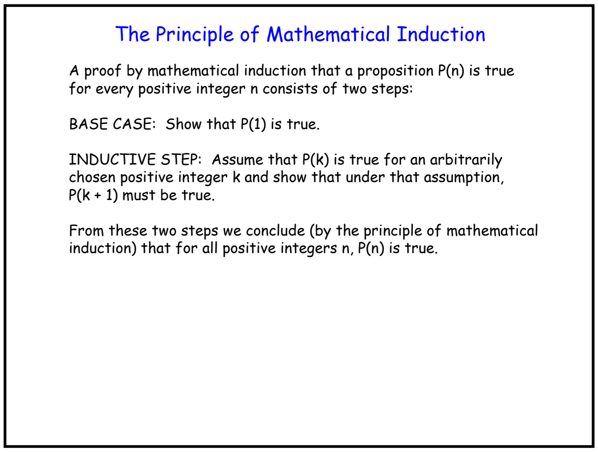

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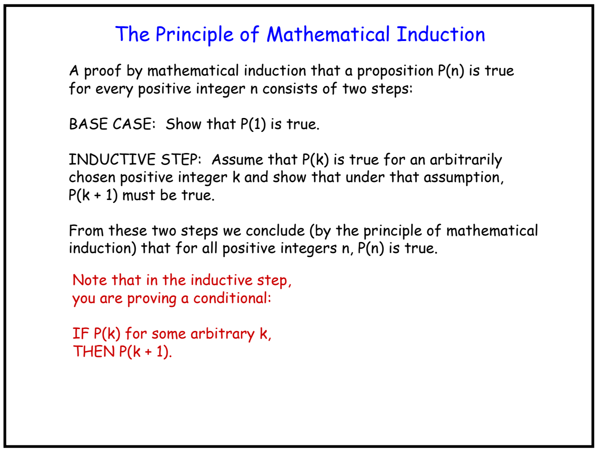

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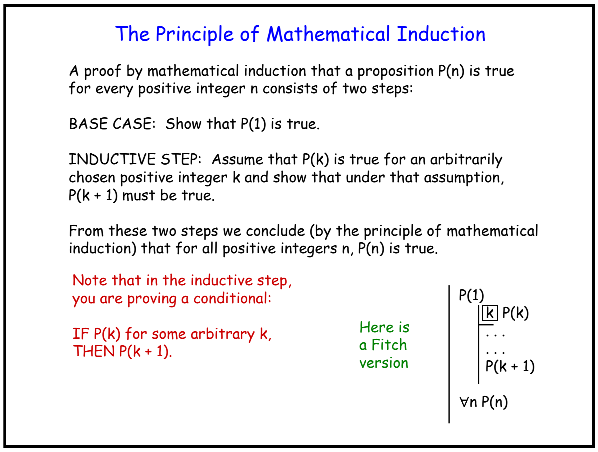

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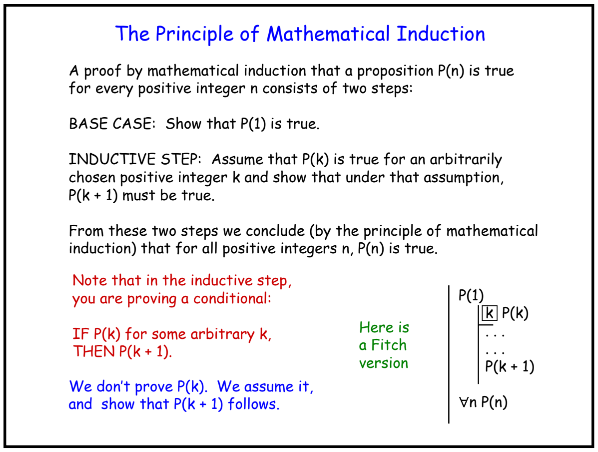

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We don't prove P(k). We assume it, and show that P(k + 1) follows.

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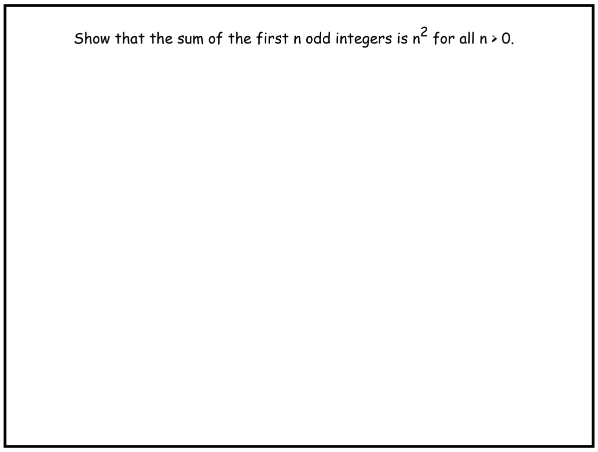
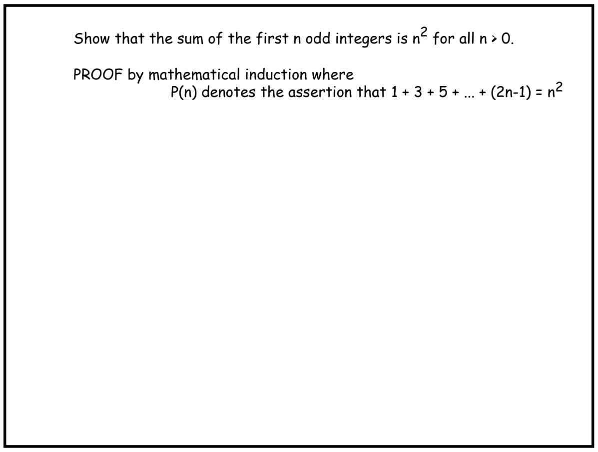


Show that the sum of the first n odd integers is n^2 for all n > 0.



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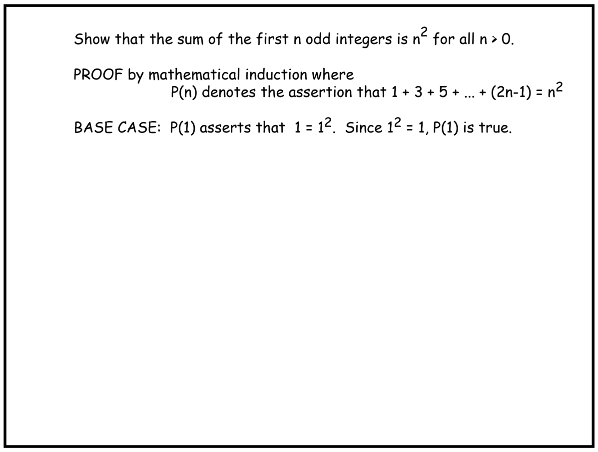
P(n) denotes the assertion that $1 + 3 + 5 + \ldots + (2n-1) = n^2$



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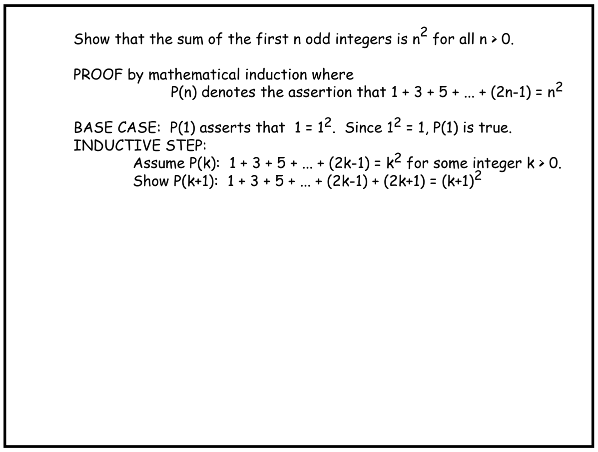

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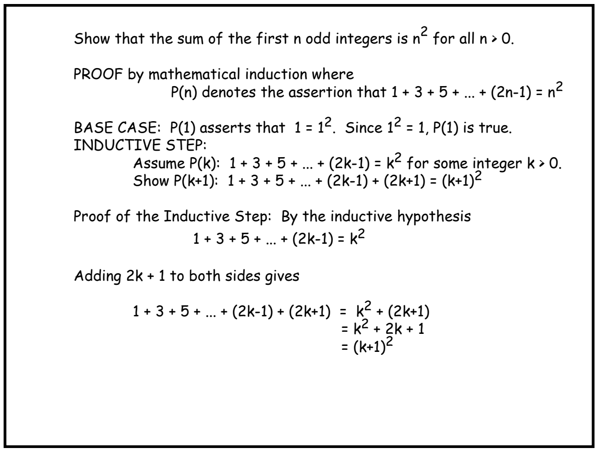
Assume P(k): $1 + 3 + 5 + \ldots + (2k-1) = k^2$ for some integer k > 0.
Show P(k+1): $1 + 3 + 5 + \ldots + (2k-1) + (2k+1) = (k+1)^2$



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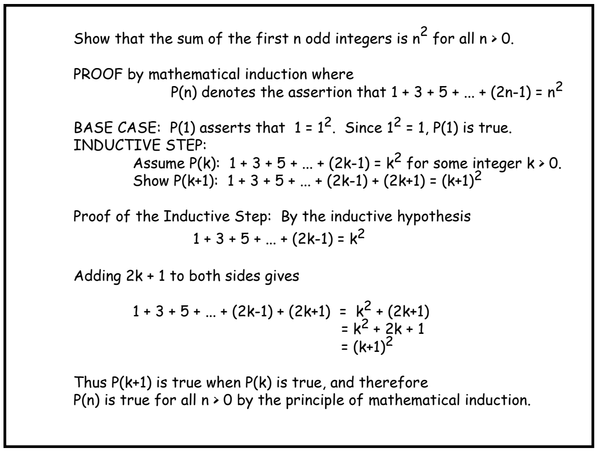
Proof of the Inductive Step: By the inductive hypothesis

$$1 + 3 + 5 + \ldots + (2k-1) = k^2$$

Adding 2k + 1 to both sides gives

$$\begin{aligned} 1 + 3 + 5 + \ldots + (2k-1) + (2k+1) &= k^2 + (2k+1) \\ &= k^2 + 2k + 1 \\ &= (k+1)^2 \end{aligned}$$



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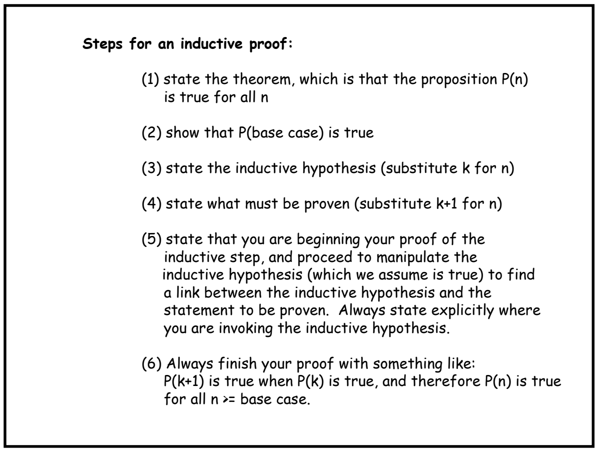
Thus P(k+1) is true when P(k) is true, and therefore
P(n) is true for all n > 0 by the principle of mathematical induction.



Steps for an inductive proof:

(1) state the theorem, which is that the proposition P(n) is true for all n

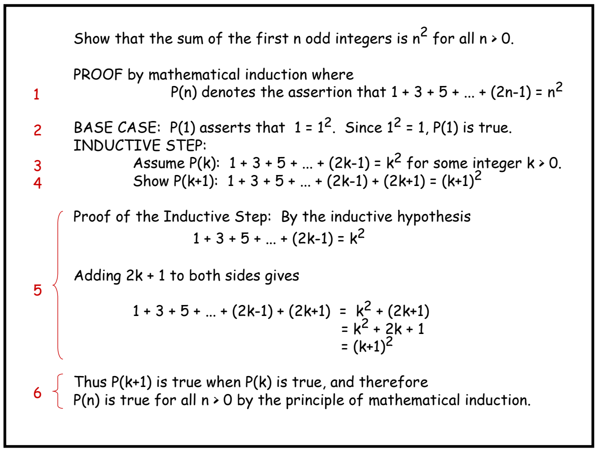
(2) show that P(base case) is true

(3) state the inductive hypothesis (substitute k for n)

(4) state what must be proven (substitute k+1 for n)

(5) state that you are beginning your proof of the inductive step, and proceed to manipulate the inductive hypothesis (which we assume is true) to find a link between the inductive hypothesis and the statement to be proven. Always state explicitly where you are invoking the inductive hypothesis.

(6) Always finish your proof with something like:
P(k+1) is true when P(k) is true, and therefore P(n) is true for all n >= base case.



Show that the sum of the first n odd integers is n^2 for all n > 0.

PROOF by mathematical induction where
1 P(n) denotes the assertion that $1 + 3 + 5 + \ldots + (2n-1) = n^2$

2 BASE CASE: P(1) asserts that $1 = 1^2$. Since $1^2 = 1$, P(1) is true.
INDUCTIVE STEP:
3 Assume P(k): $1 + 3 + 5 + \ldots + (2k-1) = k^2$ for some integer k > 0.
4 Show P(k+1): $1 + 3 + 5 + \ldots + (2k-1) + (2k+1) = (k+1)^2$

Proof of the Inductive Step: By the inductive hypothesis

$$1 + 3 + 5 + \ldots + (2k-1) = k^2$$

5 Adding 2k + 1 to both sides gives

$$\begin{aligned} 1 + 3 + 5 + \ldots + (2k-1) + (2k+1) &= k^2 + (2k+1) \\ &= k^2 + 2k + 1 \\ &= (k+1)^2 \end{aligned}$$

6 Thus P(k+1) is true when P(k) is true, and therefore
P(n) is true for all n > 0 by the principle of mathematical induction.

Prove that for n > 0, the sum of the first n integers is $\frac{n (n + 1)}{2}$

PROOF by mathematical induction, where

P(n) is $\sum_{i = 1}^{n} i = \frac{n (n + 1)}{2}$

BASE CASE: P(1) asserts that $1 = \frac{1 (1 + 1)}{2}$ which is true.

Doing the Inductive Step

Assume P(k): $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$

Show P(k+1): $\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$

Doing the Inductive Step

Assume P(k): $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$

Show P(k+1): $\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$

Proof: $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$ I.H.

$\sum_{i = 1}^{k} i + (k + 1) = \frac{k (k + 1) + 2 (k + 1)}{2}$ Add (k + 1) to both sides

$\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$ Rewrite LHS
Factor RHS

Doing the Inductive Step

Assume P(k): $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$

Show P(k+1): $\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$

Proof: $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$ $LHS_k = RHS_k$

$\sum_{i = 1}^{k} i + (k + 1) = \frac{k (k + 1) + 2 (k + 1)}{2}$ $LHS_k' = RHS_k'$

$\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$ $LHS_{k+1} = RHS_{k+1}$

Doing the Inductive Step (Alternate Approach)

Assume P(k): $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$

Show P(k+1): $\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$

Proof: $\sum_{i = 1}^{k+1} i =$

Doing the Inductive Step (Alternate Approach)

Assume P(k): $\sum_{i = 1}^{k} i = \frac{k (k + 1)}{2}$

Show P(k+1): $\sum_{i = 1}^{k+1} i = \frac{(k + 1) (k + 2)}{2}$

Proof: $\sum_{i = 1}^{k+1} i = \sum_{i = 1}^{k} i + (k + 1)$ Rewrite LHS of P(k+1) using def. of Σ

Doing the Inductive Step (Alternate Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$ Rewrite LHS of P(k+1) using def. of Σ

$= \frac{k(k+1) + 2(k+1)}{2}$ Substitute for summation using I.H., use common denominator

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Doing the Inductive Step (Alternate Approach)

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$= \frac{k(k+1) + 2(k+1)}{2}$ Substitute for summation using I.H., use common denominator

$= \frac{(k+1)(k+2)}{2}$ Algebra

QED

Doing the Inductive Step (Alternate Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$ LHS_{k+1} = equiv. expr. that exposes Inductive Hyp.

$= \frac{k(k+1) + 2(k+1)}{2}$ LHS_{k+1} = equiv. expr. using Ind. Hyp.

$= \frac{(k+1)(k+2)}{2}$ $\text{LHS}_{k+1} = \text{RHS}_{k+1}$

Doing the Inductive Step (Another Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

.
.
.

Doing the Inductive Step (Another Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

.
.
.

WRONG!!!
This is what you are trying to show.

Doing the Inductive Step (Yet Another Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

.
.
.

$X = X$ Does this prove it?

Doing the Inductive Step (Yet Another Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

$$\begin{array}{ccc} \Big\downarrow & \vdots & \Big\uparrow \\ X & = & X \end{array}$$

This is the proof

Doing the Inductive Step (Yet Another Approach)

Assume P(k): $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$

Show P(k+1): $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

Proof: $\sum_{i=1}^{k+1} i =$

$$\begin{array}{c} \Big\downarrow \\ X \\ \Big\downarrow \\ \frac{(k+1)(k+2)}{2} \end{array}$$

This is the proof

Prove: For any $n > 0$, $n(n+1)$ is even.

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P(n):

BASE CASE:

INDUCTIVE STEP:

Prove: For any $n > 0$, $n(n+1)$ is even.

P(n):

BASE CASE:

INDUCTIVE STEP:

No...WAIT!!!

Prove: For any $n > 0$, $n(n+1)$ is even.

P(n):

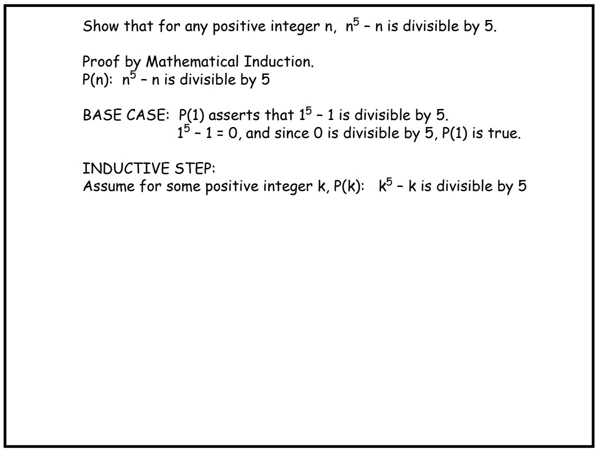
BASE CASE:

INDUCTIVE STEP:

No...WAIT!!!

One of n and n + 1 is even so n (n + 1) is even.

Induction is not needed or desired!

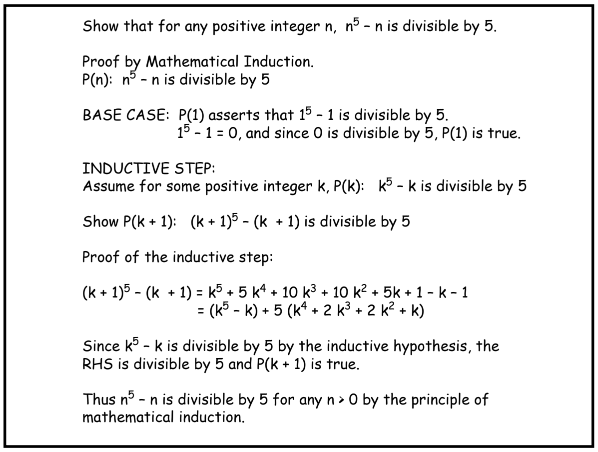


Show that for any positive integer n, $n^5 - n$ is divisible by 5.

Proof by Mathematical Induction.
P(n): $n^5 - n$ is divisible by 5

BASE CASE: P(1) asserts that $1^5 - 1$ is divisible by 5.
$1^5 - 1 = 0$, and since 0 is divisible by 5, P(1) is true.

INDUCTIVE STEP:
Assume for some positive integer k, P(k): $k^5 - k$ is divisible by 5



Show that for any positive integer n, $n^5 - n$ is divisible by 5.

Proof by Mathematical Induction.
P(n): $n^5 - n$ is divisible by 5

BASE CASE: P(1) asserts that $1^5 - 1$ is divisible by 5.
$1^5 - 1 = 0$, and since 0 is divisible by 5, P(1) is true.

INDUCTIVE STEP:
Assume for some positive integer k, P(k): $k^5 - k$ is divisible by 5

Show P(k + 1): $(k + 1)^5 - (k + 1)$ is divisible by 5

Proof of the inductive step:

$$\begin{aligned}(k + 1)^5 - (k + 1) &= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1 - k - 1 \\ &= (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)\end{aligned}$$

Since $k^5 - k$ is divisible by 5 by the inductive hypothesis, the RHS is divisible by 5 and P(k + 1) is true.

Thus $n^5 - n$ is divisible by 5 for any $n > 0$ by the principle of mathematical induction.