Midterm Review Session

The following problems will be discussed in the Midterm review session, Tuesday, Oct. 21, from 9:30 – 10:45 am in Gates B03. You may want to look over the problems before the session.

The session will be available on Stanford Online. Solutions will be available on the website sometime after the session.

Possibilty and Impossibility

1. For each of the following first-order logic sentences, circle ALL descriptions of the sentence which are accurate. Please interpret all predicates to have precisely the logical meaning they have in Tarski's World.

1) $\forall x(\text{Large}(x) \wedge \text{Dodec}(x)) \wedge [\neg\exists x\text{Large}(x) \vee \neg\exists x\text{Dodec}(x)]$

Tautology	First-Order Validity	Logical Truth
Tautological Possibility	First-Order Possibility	Logical Possibility
Tautological Impossibility	First-Order Impossibility	Logical Impossibility

2) $[\forall x(\text{Cube}(x) \rightarrow x=c)] \rightarrow \text{Cube}(c)$

Tautology	First-Order Validity	Logical Truth
Tautological Possibility	First-Order Possibility	Logical Possibility
Tautological Impossibility	First-Order Impossibility	Logical Impossibility

3)$\exists x\text{Tet}(x) \vee \forall x\neg\text{Tet}(x)$

Tautology	First-Order Validity	Logical Truth
Tautological Possibility	First-Order Possibility	Logical Possibility
Tautological Impossibility	First-Order Impossibility	Logical Impossibility

4) $\forall x\forall y(\text{SameShape}(x, y) \leftrightarrow \text{FrontOf}(x, y))$

Tautology	First-Order Validity	Logical Truth
Tautological Possibility	First-Order Possibility	Logical Possibility
Tautological Impossibility	First-Order Impossibility	Logical Impossibility

Equivalences

2. Each of the sentences in the left column has a sentence in the right column that is a First Order Equivalence. Show these equivalences by writing the sentence numbers from the left in front of the equivalent sentences on the right.

1) $\neg\exists x\exists y(P(x) \wedge \neg Q(x))$ ____ $\forall x\exists y(Q(x) \rightarrow P(x))$

2) $\forall x\forall y(P(x) \vee Q(x))$ ____ $\exists x\exists y(Q(x) \rightarrow P(x))$

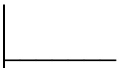
3) $\neg\exists x\neg\exists y(Q(x) \rightarrow P(x))$ ____ $\forall x\forall y(P(x) \rightarrow Q(x))$

4) $\neg\forall x\neg\exists y(\neg P(x) \rightarrow \neg Q(x))$ ____ $\neg\exists x\exists y(\neg P(x) \wedge \neg Q(x))$

Conditionals

3. Prove or disprove the following. If you prove it, write up a Fitch-style proof with all steps included. Number your steps and refer to those numbers in the reasons you give for each step. If the argument is not valid, describe in detail where the argument fails.

You may **not** use Taut Con in your proof except for the law of the excluded middle. Note: the fact that there is nothing above the line below indicates that there are no premises for this proof.



$(Q \rightarrow R) \rightarrow [(P \rightarrow Q) \rightarrow (P \rightarrow R)]$

Translations

4. Translate each of the following sentences into first-order logic. The questions describe events occurring at a Halloween party. The domain of discourse includes exactly the people who attended the party. You may only use the following predicates:

M(x)	x wore a mask to the party.
S(x, y)	x scared y at the party.
F(x, y)	x and y are friends [Note: F is symmetric, i.e. $F(x, y) \leftrightarrow F(y, x)$]

1) Nobody who didn't wear a mask scared a friend.

2) A person who wore a mask and who had no friends scared everybody else at the party.

3) No two people wearing masks scared each other.

4) Everybody with at least two friends scared somebody.

Formal Proofs with Quantifiers

5. Give a Fitch-style proof for the following. Number your steps and refer to those numbers in the reasons you give for each step. You **are allowed** to use Taut Con to justify proof steps involving only Boolean connectives.

$\forall x\ (\neg\ (D(x) \rightarrow \neg A(x)))$
$\forall x\ (A(x) \rightarrow \exists y\ B(y))$
$\exists x\ A(x)$

$\exists x\ (B(x) \wedge D(x))$