

CS103A

10/20/08

Review Session

Tues., Oct. 21
 9:30 - 10:45 am
 Location: **Gates B03**
 Stanford Online, but
 not broadcast

Midterm Exam

Thurs., Oct. 23
 7 - 9 pm
 Location: **Gates B01**
 Open book (LPL),
 Open Notes, Crib Sheet

Tomorrow!

SCPD Students

We request that Bay Area SCPD students take the midterm on campus Thursday night.

Please let us know if that will work for you or whether you will need an alternate arrangement.

We also request that non-Bay Area students let us know where you are so that we can be sure to get the exam to you on time.

Universal Introduction

a	P(a)
⋮	⋮
Q	Q

$\forall xP(x)$ \forall Intro

General Conditional Proof

a	P(a)
⋮	⋮
Q(a)	Q(a)

$\forall x(P(x) \rightarrow Q(x))$ \forall Intro

Existential Elimination

$\exists xP(x)$	P(a)
⋮	⋮
Q	Q

\exists Elim

Reminder: the boxed constants cannot occur outside the subproofs where they are introduced.

Universal Elimination

$\forall xP(x)$	P(a)
⋮	⋮
P(a)	P(a)

\forall Intro

Existential Introduction

P(a)	P(a)
⋮	⋮
$\exists xP(x)$	$\exists xP(x)$

\exists Intro

1. $\forall x$ Cube(x)	
2. Cube(a)	\forall Elim 1
3. $\exists x$ Cube(x)	\exists Intro 2

1. $\forall x$ Cube(x)	
2. \forall	
3. Cube(a)	\forall Elim 2
4. $\exists x$ Cube(x)	\exists Intro 3
5. $\forall x$ Cube(x) \rightarrow $\exists x$ Cube(x)	\rightarrow Intro 2-4
6. $\exists x$ Cube(x)	\rightarrow Elim 1, 5

1. $\forall x$ Cube(x)	
2. \forall	
3. Cube(a)	\forall Elim 2
4. $\exists x$ Cube(x)	\exists Intro 3
5. $\forall y$ $\exists x$ Cube(x)	\forall Intro 2-4
6. $\exists x$ Cube(x)	\forall Elim 5

<ol style="list-style-type: none"> 1. $\forall x \text{ Cube}(x)$ 2. $\boxed{a} \nabla$ 3. $\text{Cube}(a)$ 4. $\exists x \text{ Cube}(x)$ 5. $\forall y \exists x \text{ Cube}(x)$ 6. $\exists x \text{ Cube}(x)$ 	\forall Elim 2 \exists Intro 3 \forall Intro 2-4 \forall Elim 5
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<ol style="list-style-type: none"> 1. $\forall x \text{ Cube}(x)$ 2. $\text{Cube}(a)$ 3. $\exists x \text{ Cube}(x)$ 	\forall Elim 1 \exists Intro 2
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13.3

$\forall x \text{ Cube}(x)$ $\forall x \text{ Small}(x)$	$\forall x(\text{Cube}(x) \wedge \text{Small}(x))$
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13.3

<ol style="list-style-type: none"> 1. $\forall x \text{ Cube}(x)$ 2. $\forall x \text{ Small}(x)$ 3. $\text{Cube}(a)$ 4. $\text{Small}(a)$ 5. $\text{Cube}(a) \wedge \text{Small}(a)$ 6. $\forall x(\text{Cube}(x) \wedge \text{Small}(x))$ 	\forall Elim 1 \forall Elim 2 \wedge Intro 3,4 \forall Intro
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13.3

<ol style="list-style-type: none"> 1. $\forall x \text{ Cube}(x)$ 2. $\forall x \text{ Small}(x)$ 3. $\text{Cube}(a)$ 4. $\text{Small}(a)$ 5. $\text{Cube}(a) \wedge \text{Small}(a)$ 6. $\forall x(\text{Cube}(x) \wedge \text{Small}(x))$ 	\forall Elim 1 \forall Elim 2 \wedge Intro 3,4 \forall Intro
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This won't work!

 You have to cite a subproof

13.3

<ol style="list-style-type: none"> 1. $\forall x \text{ Cube}(x)$ 2. $\forall x \text{ Small}(x)$ 3. $\boxed{a} \nabla$ 4. $\text{Cube}(a)$ 5. $\text{Small}(a)$ 6. $\text{Cube}(a) \wedge \text{Small}(a)$ 7. $\forall x(\text{Cube}(x) \wedge \text{Small}(x))$ 	\forall Elim 1 \forall Elim 2 \wedge Intro 4,5 \forall Intro 3 - 6
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$\forall x \neg \text{Cube}(x)$	$\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$
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1. $\forall x \neg \text{Cube}(x)$
 2. $\boxed{a} \nabla$
 3.
 4.
 5.
 6.
 7. $\neg(\text{Cube}(a) \wedge \text{Small}(a))$
 8. $\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$ \forall Intro 2-7

1. $\forall x \neg \text{Cube}(x)$
 2. $\boxed{a} \nabla$
 3. $\text{Cube}(a) \wedge \text{Small}(a)$
 4. \perp
 5. $\neg(\text{Cube}(a) \wedge \text{Small}(a))$ \neg Intro 3-6
 6. $\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$ \forall Intro 2-7

1. $\forall x \neg \text{Cube}(x)$
 2. $\boxed{a} \nabla$
 3. $\text{Cube}(a) \wedge \text{Small}(a)$
 4. $\text{Cube}(a)$ \wedge Elim 3
 5. $\neg \text{Cube}(a)$ \forall Elim 1
 6. \perp \perp Intro 4,5
 7. $\neg(\text{Cube}(a) \wedge \text{Small}(a))$ \neg Intro 3-6
 8. $\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$ \forall Intro 2-7

Alternate Approach to Previous Proof

1. $\forall x \neg \text{Cube}(x)$
 2. $\boxed{a} \nabla$
 3. $\neg \text{Cube}(a)$ \forall Elim 1
 4.
 5.
 6. $\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$

Alternate Approach to Previous Proof

1. $\forall x \neg \text{Cube}(x)$
 2. $\boxed{a} \nabla$
 3. $\neg \text{Cube}(a)$ \forall Elim 1
 4. $\neg \text{Cube}(a) \vee \neg \text{Small}(a)$ \vee Intro 2
 5. $\neg(\text{Cube}(a) \wedge \text{Small}(a))$ Taut Con 4
 6. $\forall x \neg(\text{Cube}(x) \wedge \text{Small}(x))$ \forall Intro 2-5

13.24

$\exists x (\text{Cube}(x) \wedge \text{Small}(x))$

$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$

13.24

	1.	$\exists x (\text{Cube}(x) \wedge \text{Small}(x))$	
	2.	$\boxed{a} \nabla \text{Cube}(a) \wedge \text{Small}(a)$	
	3.	...	
	4.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\exists Elim

13.24

	1.	$\exists x (\text{Cube}(x) \wedge \text{Small}(x))$	
	2.	$\boxed{a} \nabla \text{Cube}(a) \wedge \text{Small}(a)$	
	3.		
	4.		
	5.		
	6.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	
	7.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\exists Elim 2-7

13.24

	1.	$\exists x (\text{Cube}(x) \wedge \text{Small}(x))$	
	2.	$\boxed{a} \nabla \text{Cube}(a) \wedge \text{Small}(a)$	
	3.	$\text{Cube}(a)$	\wedge Elim 2
	4.	$\exists x \text{Cube}(x)$	\exists Intro 3
	5.		
	6.	$\exists x \text{Small}(x)$	
	7.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\wedge Intro 4, 6
	8.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\exists Elim 2-7

13.24

	1.	$\exists x (\text{Cube}(x) \wedge \text{Small}(x))$	
	2.	$\boxed{a} \nabla \text{Cube}(a) \wedge \text{Small}(a)$	
	3.	$\text{Cube}(a)$	\wedge Elim 2
	4.	$\exists x \text{Cube}(x)$	\exists Intro 3
	5.	$\text{Small}(a)$	\wedge Elim 2
	6.	$\exists x \text{Small}(x)$	\exists Intro 5
	7.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\wedge Intro 4, 6
	8.	$\exists x \text{Cube}(x) \wedge \exists x \text{Small}(x)$	\exists Elim 2-7

	1.	$\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$	
	2.		
	3.		
	4.		
	5.		
	6.		
	7.		
	8.		
	9.		
	10.	$\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$	
	11.	$\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$	\forall Intro 2-10

	1.	$\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$	
	2.	$\boxed{c} \nabla \text{Cube}(c)$	
	3.		
	4.		
	5.		
	6.		
	7.		
	8.		
	9.		
	10.	$\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$	
	11.	$\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$	\forall Intro 2-10

1. $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$

2. $\boxed{c} \nabla \text{Cube}(c)$

3. $\boxed{b} \nabla \text{Tet}(b) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

4. $\forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

5. $\text{Cube}(c) \rightarrow \text{Smaller}(c, b)$

6.

7.

8. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$

9.

10. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Elim 1, 3-9

11. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$ \forall Intro 2-10

1. $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$

2. $\boxed{c} \nabla \text{Cube}(c)$

3. $\boxed{b} \nabla \text{Tet}(b) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

4. $\forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$ \wedge Elim 3

5.

6.

7.

8. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$

9.

10. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Elim 1, 3-9

11. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$ \forall Intro 2-10

1. $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$

2. $\boxed{c} \nabla \text{Cube}(c)$

3. $\boxed{b} \nabla \text{Tet}(b) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

4. $\forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$ \wedge Elim 3

5. $\text{Cube}(c) \rightarrow \text{Smaller}(c, b)$ \forall Elim 4

6.

7.

8. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$

9.

10. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Elim 1, 3-9

11. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$ \forall Intro 2-10

1. $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$

2. $\boxed{c} \nabla \text{Cube}(c)$

3. $\boxed{b} \nabla \text{Tet}(b) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

4. $\forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$ \wedge Elim 3

5. $\text{Cube}(c) \rightarrow \text{Smaller}(c, b)$ \forall Elim 4

6. $\text{Smaller}(c, b)$ \rightarrow Elim 2, 5

7.

8. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$

9.

10. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Elim 1, 3-9

11. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$ \forall Intro 2-10

1. $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, y)))$

2. $\boxed{c} \nabla \text{Cube}(c)$

3. $\boxed{b} \nabla \text{Tet}(b) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$

4. $\forall x (\text{Cube}(x) \rightarrow \text{Smaller}(x, b))$ \wedge Elim 3

5. $\text{Cube}(c) \rightarrow \text{Smaller}(c, b)$ \forall Elim 4

6. $\text{Smaller}(c, b)$ \rightarrow Elim 2, 5

7. $\text{Tet}(b)$ \wedge Elim 3

8. $\text{Tet}(b) \wedge \text{Smaller}(c, b)$ \wedge Intro 7, 6

9. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Intro 8

10. $\exists y (\text{Tet}(y) \wedge \text{Smaller}(c, y))$ \exists Elim 1, 3-9

11. $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Smaller}(x, y)))$ \forall Intro 2-10

Universal 2

$\forall x (\text{Tet}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(x, y)))$

$\forall x \forall y ((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{LeftOf}(x, y))$

Universal 2

1. $\forall x (\text{Tet}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(x, y)))$
2. $\boxed{a\ b} \nabla \text{Tet}(a) \wedge \text{Cube}(b)$
3. $\text{Tet}(a) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$
4. $\forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$
5. $\text{Cube}(b) \rightarrow \text{LeftOf}(a, b)$
6. $\text{LeftOf}(a, b)$
7. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{LeftOf}(x, y))$ ∇ Intro 2-6

Universal 2

1. $\forall x (\text{Tet}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(x, y)))$
2. $\boxed{a\ b} \nabla \text{Tet}(a) \wedge \text{Cube}(b)$
3. $\text{Tet}(a) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$ ∇ Elim 1
4. $\forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$
5. $\text{Cube}(b) \rightarrow \text{LeftOf}(a, b)$
6. $\text{LeftOf}(a, b)$
7. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{LeftOf}(x, y))$ ∇ Intro 2-6

Universal 2

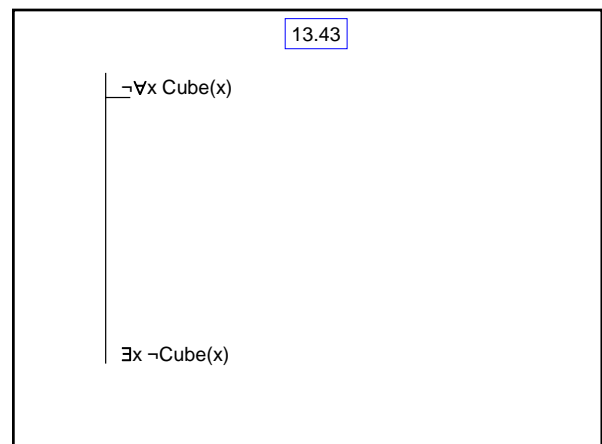
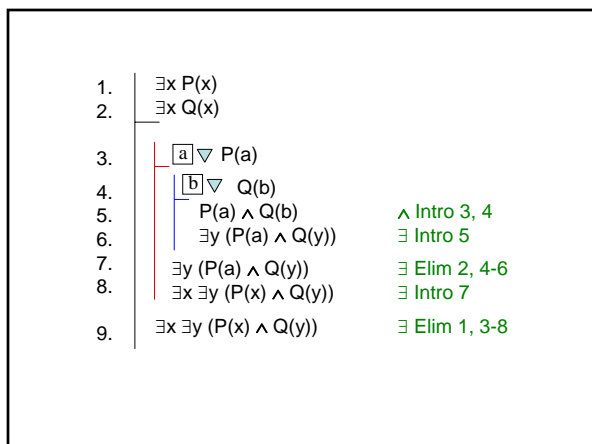
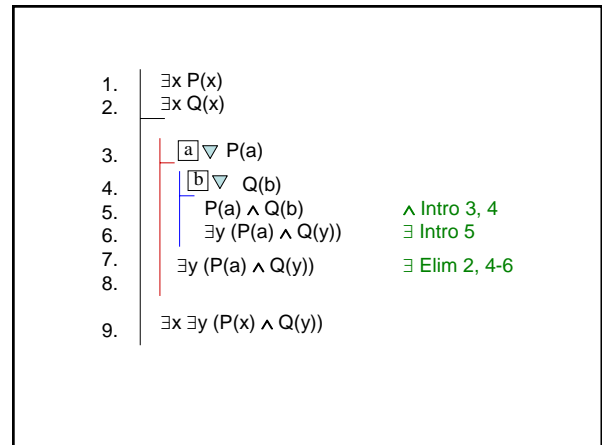
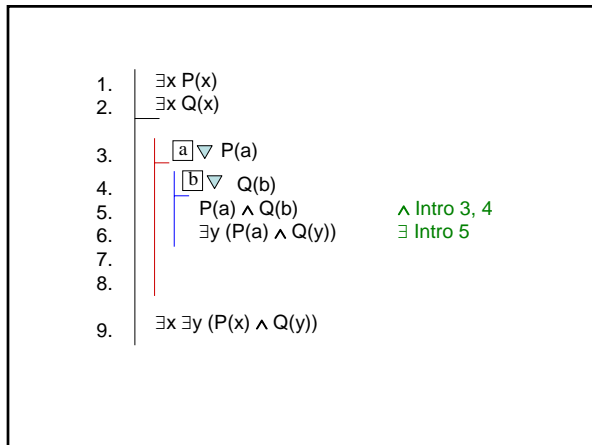
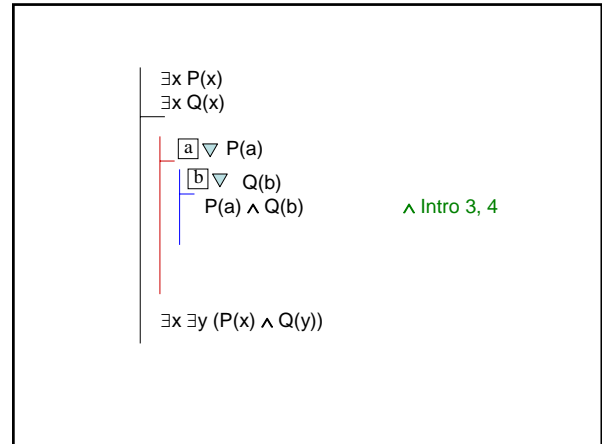
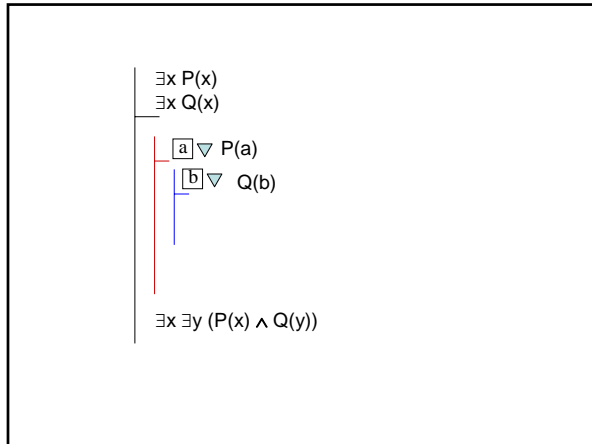
1. $\forall x (\text{Tet}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(x, y)))$
2. $\boxed{a\ b} \nabla \text{Tet}(a) \wedge \text{Cube}(b)$
3. $\text{Tet}(a) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$ ∇ Elim 1
4. $\forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$ Taut Con 2, 3
5. $\text{Cube}(b) \rightarrow \text{LeftOf}(a, b)$
6. $\text{LeftOf}(a, b)$
7. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{LeftOf}(x, y))$ ∇ Intro 2-6

Universal 2

1. $\forall x (\text{Tet}(x) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(x, y)))$
2. $\boxed{a\ b} \nabla \text{Tet}(a) \wedge \text{Cube}(b)$
3. $\text{Tet}(a) \rightarrow \forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$ ∇ Elim 1
4. $\forall y (\text{Cube}(y) \rightarrow \text{LeftOf}(a, y))$ Taut Con 2, 3
5. $\text{Cube}(b) \rightarrow \text{LeftOf}(a, b)$ ∇ Elim 4
6. $\text{LeftOf}(a, b)$ Taut Con 2, 6
7. $\forall x \forall y ((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{LeftOf}(x, y))$ ∇ Intro 2-6

$$\frac{\exists x P(x) \quad \exists x Q(x)}{\exists x \exists y (P(x) \wedge Q(y))}$$

$$\frac{\exists x P(x) \quad \exists x Q(x) \quad \boxed{a} \nabla P(a)}{\exists x \exists y (P(x) \wedge Q(y))}$$



13.43

1.	$\neg \forall x \text{ Cube}(x)$		
2.	$\neg \exists x \neg \text{Cube}(x)$		
3.			
4.			
5.			
6.			
7.			
8.	$\forall x \text{ Cube}(x)$		
9.	\perp	\perp Intro 1, 8	
10.	$\exists x \neg \text{Cube}(x)$	\neg Intro 2-9	

13.43

1.	$\neg \forall x \text{ Cube}(x)$		
2.	$\neg \exists x \neg \text{Cube}(x)$		
3.	$a \nabla$		
4.			
5.			
6.			
7.	$\text{Cube}(a)$		
8.	$\forall x \text{ Cube}(x)$	\forall Intro 3-7	
9.	\perp	\perp Intro 1, 8	
10.	$\exists x \neg \text{Cube}(x)$	\neg Intro 2-9	

13.43

1.	$\neg \forall x \text{ Cube}(x)$		
2.	$\neg \exists x \neg \text{Cube}(x)$		
3.	$a \nabla$		
4.	$\neg \text{Cube}(a)$		
5.			
6.	\perp	\perp Intro	
7.	$\text{Cube}(a)$	\neg Intro 4-6	
8.	$\forall x \text{ Cube}(x)$	\forall Intro 3-7	
9.	\perp	\perp Intro 1, 8	
10.	$\exists x \neg \text{Cube}(x)$	\neg Intro 2-9	

13.43

1.	$\neg \forall x \text{ Cube}(x)$		
2.	$\neg \exists x \neg \text{Cube}(x)$		
3.	$a \nabla$		
4.	$\neg \text{Cube}(a)$		
5.	$\exists x \neg \text{Cube}(x)$	\exists Intro 4	
6.	\perp	\perp Intro 2, 5	
7.	$\text{Cube}(a)$	\neg Intro 4-6	
8.	$\forall x \text{ Cube}(x)$	\forall Intro 3-7	
9.	\perp	\perp Intro 1, 8	
10.	$\exists x \neg \text{Cube}(x)$	\neg Intro 2-9	

13.43

$\neg \forall x \text{ Cube}(x)$		
$\exists x \neg \text{Cube}(x)$	\checkmark FO Con	
$\neg(\text{Cube}(a) \wedge \text{Tet}(b))$		
$\neg \text{Cube}(a) \vee \neg \text{Tet}(b)$	\checkmark Taut Con	

13.43

$\neg \forall x \text{ Cube}(x)$		
$\exists x \neg \text{Cube}(x)$	\checkmark FO Con	
$\neg(\text{Cube}(a) \wedge \text{Tet}(b))$		
$\neg \text{Cube}(a) \vee \neg \text{Tet}(b)$	\checkmark Taut Con	
$\neg(\exists x \text{ Cube}(x) \wedge \exists x \text{ Tet}(x))$		
$\neg \exists x \text{ Cube}(x) \vee \neg \exists x \text{ Tet}(x)$	\checkmark Taut Con	