

Complete Inference Rules

<p style="text-align: center;"><u>= Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P(n)$ \dots $n = m$ \dots $P(m)$ </div>	<p style="text-align: center;"><u>= Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> \dots $a = a$ \dots </div>	<p style="text-align: center;"><u>^ Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P1 \wedge P2$ \dots $P1$ $P2$ </div>	<p style="text-align: center;"><u>^ Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P1$ \dots $P2$ \dots $P1 \wedge P2$ </div>
<p style="text-align: center;"><u>v Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> P \dots $P \vee Q$ </div>	<p style="text-align: center;"><u>v Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P1 \vee P2$ <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> $P1$ \dots S </div> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> $P2$ \dots S </div> S </div>	<p style="text-align: center;"><u>¬ Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $\neg\neg P$ P </div>	<p style="text-align: center;"><u>¬ Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> P \dots \perp </div> $\neg P$ </div>
<p style="text-align: center;"><u>⊥ Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> P \dots $\neg P$ \perp </div>	<p style="text-align: center;"><u>⊥ Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> \perp \dots P </div>	<p style="text-align: center;"><u>↔ Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> P \dots Q </div> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> Q \dots P </div> $P \leftrightarrow Q$ </div>	<p style="text-align: center;"><u>↔ Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P \leftrightarrow Q$ (or $Q \leftrightarrow P$) \dots P \dots Q </div>
<p style="text-align: center;"><u>→ Intro</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 2px;"> P \dots Q </div> $P \rightarrow Q$ </div>	<p style="text-align: center;"><u>→ Elim</u></p> <div style="border-left: 1px solid black; border-right: 1px solid black; padding: 5px;"> $P \rightarrow Q$ P \dots Q </div>		

Rules for Quantifiers

<p><u>Universal Elimination</u></p> $\left \begin{array}{l} \forall xP(x) \\ \cdot \\ \cdot \\ \cdot \\ P(a) \end{array} \right. \quad \forall \text{ Elim}$	<p><u>Universal Introduction</u></p> $\left \begin{array}{l} \boxed{a} \\ \cdot \\ \cdot \\ \cdot \\ P(a) \end{array} \right. \quad \forall \text{ Intro}$ $\forall xP(x)$
<p><u>General Conditional Proof</u></p>	
$\left \begin{array}{l} \boxed{a} P(a) \\ \cdot \\ \cdot \\ \cdot \\ Q(a) \end{array} \right. \quad \forall \text{ Intro}$ $\forall x(P(x) \rightarrow Q(x))$	

<p><u>Existential Introduction</u></p> $\left \begin{array}{l} P(a) \\ \cdot \\ \cdot \\ \cdot \\ \exists xP(x) \end{array} \right. \quad \exists \text{ Intro}$	<p><u>Existential Elimination</u></p> $\left \begin{array}{l} \exists xP(x) \\ \boxed{a} P(a) \\ \cdot \\ \cdot \\ \cdot \\ Q \end{array} \right. \quad \exists \text{ Elim}$ <p style="color: blue; margin-left: 100px;">Q does not contain a</p>
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Remember that boxed constants cannot occur outside their subproofs.