Handout #24 CS103A
October 17, 2008 Robert Plummer

Practice Midterm

Exam Rules:

1) You have 2.0 hours to complete this exam.
2) Do not include your scratch work with your exam. Please work the solutions out on another sheet of paper and then write your solutions neatly on the exam.
3) **Be certain that you submit only one answer to each question.** If you happen to write out more than one solution to a question, indicate clearly which one we should grade; otherwise, we will make an arbitrary choice.
4) This is an open-book (LPL only), open-note exam. You may also use a one page cheat-sheet (both sides OK) of your own design. You will find the inference rules at the back of the exam.
5) Please write clearly and neatly. Using a pencil works best

The questions below are from the Autumn 2007 Midterm. We don't promise that your questions will be of exactly the same form, but this exam gives a good idea of what to expect.

Trying to do this exam in a two-hour period is the best preparation. Solutions will be distributed next week.

1. Tautology, First-Order Validity, and Logical Truth (18 points)

For each of the following sentences, circle the **correct** description:

(1) $(\exists x\ Cube(x) \rightarrow \forall x\ Cube(x)) \vee (\forall x\ Cube(x) \rightarrow \exists x\ Cube(x))$

(a) Tautology
(b) First-Order Valid, but not tautology
(c) Logical Truth, but not First-Order valid
(d) Not Logical Truth

(2) $(\neg\exists x\ LessThan(x,x)) \wedge (\forall x(x=x))$

(a) Tautology
(b) First-Order Valid, but not tautology
(c) Logical Truth, but not First-Order valid
(d) Not Logical Truth

(3) $(Cube(c) \rightarrow \exists x\ Cube(x)) \wedge (Cube(d) \leftrightarrow \forall x\ Cube(d))$

(a) Tautology
(b) First-Order Valid, but not tautology
(c) Logical Truth, but not First-Order valid
(d) Not Logical Truth

(4) $(Cube(c) \wedge \neg Small(c)) \wedge (\neg Cube(c) \vee Small(c))$

(a) Tautology
(b) Logical Truth, but not tautology
(c) Not Logical Truth, but tautologically possible
(d) Tautologically Impossible

(5) $\neg\forall x\forall y((Cube(x) \wedge Tet(y)) \rightarrow Larger(x,y)) \leftrightarrow \exists x\exists y(Cube(x) \wedge Tet(y) \wedge \neg Larger(x,y))$

(a) Tautology
(b) First-Order Valid, but not tautology
(c) Not First-Order Valid, but tautologically possible
(d) Tautologically Impossible

(6) $\forall x\ ((\forall y\ P(x,y)) \rightarrow Q(x)) \leftrightarrow \forall x\forall y\ (P(x,y) \rightarrow Q(x))$

(a) Tautology
(b) First-Order Valid, but not tautology
(c) Not First-Order valid, but tautologically possible
(d) Tautologically Impossible

2. Equivalences (20 points)

For each given sentence, circle the choice that is a First-Order Equivalence.

(1) $\exists x\ (P(x) \vee Q(x))$

(a) $\forall x\ (P(x) \wedge Q(x))$
(b) $\neg(\forall x \neg P(x)) \vee \neg(\forall x \neg Q(x))$
(c) $\neg(\forall x \neg P(x)) \wedge \neg(\forall x \neg Q(x))$
(d) $\neg(\forall x \neg P(x)) \vee (\forall x \neg Q(x))$

(2) $\forall x\ (P(x) \rightarrow Q(x))$

(a) $\neg\exists x\ \neg(\neg Q(x) \rightarrow P(x))$
(b) $\neg\exists x\ (\neg Q(x) \rightarrow \neg P(x))$
(c) $\neg\exists x\ \neg(\neg Q(x) \rightarrow \neg P(x))$
(d) $\neg\exists x\ \neg(\neg Q(x) \vee \neg P(x))$

(3) $\exists x\ P(x) \rightarrow \forall y\ Q(y)$

(a) $\exists x\ \neg P(x) \vee \forall y\ Q(y)$
(b) $\exists x\ \neg P(x) \wedge \forall y\ Q(y)$
(c) $\exists x\ \neg P(x) \rightarrow \forall y\ Q(y)$
(d) $\forall x\ \neg P(x) \vee \forall y\ Q(y)$

(4) $\neg\forall x\ \exists y\ (A(y) \wedge B(x, y))$

(a) $\forall x\ \forall y\ (A(y) \rightarrow \neg B(x, y))$
(b) $\exists x\ \forall y\ (A(y) \rightarrow \neg B(x, y))$
(c) $\forall x\ \forall y\ (A(y) \rightarrow B(x, y))$
(d) $\forall x\ \exists y\ (A(y) \rightarrow B(x, y))$

3. Conditionals (20 points)

Prove or disprove the following. If you prove it, write up a Fitch-style proof with all steps included. Number your steps and refer to those numbers in the reasons you give for each step. If the argument is not valid, describe in detail where the argument fails.

You may **not** use Taut Con in your proof except for the law of excluded middle. Note: the fact that there is nothing above the Fitch bar indicates that there are no premises for this proof.

$$\mathbf{(P \rightarrow (\neg R \rightarrow \neg Q)) \rightarrow ((P \land Q) \rightarrow R)}$$

4. Translations (20 points)

Translate each of the following sentences into first-order logic. The statements describe events during a single Stanford football game. The domain of discourse is all Stanford students (including team members). You may only use the following predicates:

T(x)	x is a team member.
W(x, y)	x watched a play by y.
C(x, y)	x sat close to y [Note: C is symmetric, i.e. $C(x, y) \leftrightarrow C(y, x)$, and reflexive, i.e. C(x,x) is true]
F(x, y)	x and y are friends [Note: F is symmetric, i.e. $F(x, y) \leftrightarrow F(y, x)$, and NOT reflexive, i.e. $\neg F(x,x)$ is true]

1) Nobody watched his own play.

2) Only friends sat close to each other.
Note: Our view was that "each other" implied that we were talking about two people. Even if you thought that it was not possible to satisfy this situation (because you sit close to yourself and are not friends with yourself), you could still write down a sentence that described it. We were very lenient on the grading of this one.

3) Some student who has no friends and is not on the team sat close to no one else.

4) Everybody who watched at least 3 plays by different players is a friend of someone on the team.

5. Formal Proofs with Quantifiers (22 points)

Give a Fitch-style proof for the following. Number your steps and refer to those numbers in the reasons you give for each step. You **are allowed** to use Taut Con freely to justify proof steps where it would be accepted by the program Fitch. The comments at the side show where the sentences came from, in case that helps.

$\forall x\ (A(x) \rightarrow \exists y\ (D(y) \land C(x, y)))$;Every act is caused by a desire

$\forall x\ (D(x) \rightarrow \exists y\ (B(y) \land C(x, y)))$;Every desire is caused by a brain process

$\forall x \forall y \forall z\ ((C(x, y) \land C(y, z)) \rightarrow C(x, z))$;For all x, y, z, if x is caused by y and y is caused by z, then x is caused by z

$\forall x\ (A(x) \rightarrow \exists y\ (B(y) \land C(x, y)))$;Every act is caused by a brain process