

CS103A

10/17/08

<p>Review Session</p> <p>Monday or Tuesday TBA</p>	<p>Midterm Exam</p> <p>Thurs., Oct. 23 7 - 9 pm Location: TBA Open book (LPL), Open Notes, Crib Sheet Coverage: through 10/20</p>
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12.23

Proofs with Quantifiers

Every child is right-handed or intelligent No intelligent child eats liver There is a child who eats liver and onions	----- There is a right-handed child who eats onions
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Proofs with Quantifiers

If we know $\forall x S(x)$, and b is an object, then we can assert $S(b)$.

Tet(b) $\forall x \text{ Large}(x)$ ----- ... Large(b)	
--	--

Proofs with Quantifiers

If we know $\forall x S(x)$, and b is an object, then we can assert $S(b)$.

Tet(b) $\forall x \text{ Large}(x)$ ----- ... Large(b)	\forall Elim	$\forall x P(x)$. . . P(a) (where a refers to an object)
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Universal Elimination
 Universal Instantiation

Proofs with Quantifiers

If b is an object and we know $S(b)$, then we can assert $\exists x S(x)$.

Tet(b) ----- ... $\exists x \text{ Tet}(x)$	
--	--

Proofs with Quantifiers

If b is an object and we know $S(b)$, then we can assert $\exists x S(x)$.

Tet(b) ----- ... $\exists x \text{ Tet}(x)$	\exists Intro	P(a) . . . $\exists x P(x)$
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Existential Introduction
 Existential Generalization

1.		$\forall x (\text{Tet}(x) \rightarrow \text{Green}(x))$	
2.		$\forall x (\text{Green}(x) \rightarrow \text{Smaller}(x, a))$	
3.		$\text{Tet}(t)$	
4.			
5.			
6.			
7.			
8.			
9.		$\exists x (\text{Green}(x) \wedge \text{Smaller}(x, a))$	

1.		$\forall x (\text{Tet}(x) \rightarrow \text{Green}(x))$	
2.		$\forall x (\text{Green}(x) \rightarrow \text{Smaller}(x, a))$	
3.		$\text{Tet}(t)$	
4.		$\text{Tet}(t) \rightarrow \text{Green}(t)$	$\forall \text{Elim } 1$
5.		$\text{Green}(t) \rightarrow \text{Smaller}(t, a)$	$\forall \text{Elim } 2$
6.			
7.			
8.			
9.		$\exists x (\text{Green}(x) \wedge \text{Smaller}(x, a))$	

1.		$\forall x (\text{Tet}(x) \rightarrow \text{Green}(x))$	
2.		$\forall x (\text{Green}(x) \rightarrow \text{Smaller}(x, a))$	
3.		$\text{Tet}(t)$	
4.		$\text{Tet}(t) \rightarrow \text{Green}(t)$	$\forall \text{Elim } 1$
5.		$\text{Green}(t) \rightarrow \text{Smaller}(t, a)$	$\forall \text{Elim } 2$
6.		$\text{Green}(t)$	$\rightarrow \text{Elim } 3, 4$
7.		$\text{Smaller}(t, a)$	$\rightarrow \text{Elim } 6, 5$
8.		$\text{Green}(t) \wedge \text{Smaller}(t, a)$	$\wedge \text{Intro } 6, 7$
9.		$\exists x (\text{Green}(x) \wedge \text{Smaller}(x, a))$	$\exists \text{Intro } 8$

Proofs with Quantifiers

If we introduce b as the name of an arbitrary object in the domain of discourse, and we prove $S(b)$, then we can assert $\forall x S(x)$.

		Suppose b is an object	
		...	
		$\text{Tet}(b)$	
		$\forall x \text{Tet}(x)$	

Proofs with Quantifiers

If we introduce b as the name of an arbitrary object in the domain of discourse, and we prove $S(b)$, then we can assert $\forall x S(x)$.

		Suppose b is an object	
		...	
		$\text{Tet}(b)$	
		$\forall x \text{Tet}(x)$	
		$\forall x S(x)$	

\boxed{b} \forall
 Bounded constant

Proofs with Quantifiers

If we introduce b as the name of an arbitrary object in the domain of discourse, and we prove $S(b)$, then we can assert $\forall x S(x)$.

		Suppose b is an object	
		...	
		$\text{Tet}(b)$	
		$\forall x \text{Tet}(x)$	
		$\forall x S(x)$	

\boxed{b} \forall
 b cannot occur outside the subproof

$$\begin{array}{|l}
 \forall x (P(x) \rightarrow Q(x)) \\
 \forall z (Q(z) \rightarrow R(z)) \\
 \hline
 \boxed{a} \forall P(a) \\
 \hline
 R(a) \\
 \hline
 \forall x(P(x) \rightarrow R(x)) \quad \forall \text{Intro}
 \end{array}$$

1. $\forall x (P(x) \rightarrow Q(x))$
2. $\forall z (Q(z) \rightarrow R(z))$
3. $\boxed{a} \forall P(a)$
4. $P(a) \rightarrow Q(a)$ $\forall \text{Elim } 1$
5. $Q(a)$ $\rightarrow \text{Elim } 3, 4$
6. $Q(a) \rightarrow R(a)$ $\forall \text{Elim } 2$
7. $R(a)$ $\rightarrow \text{Elim } 5, 6$
8. $\forall x(P(x) \rightarrow R(x))$ $\forall \text{Intro } 3 - 7$

Proofs with Quantifiers

Suppose we know that $\exists x S(x)$, that we introduce c as the name of an object, and that we assume $S(c)$. If we can conclude a sentence Q not involving c , then we can conclude that Q follows from the original premises.

$$\begin{array}{|l}
 \exists x S(x) \\
 \boxed{c} S(c) \\
 \dots \\
 Q \\
 \hline
 Q
 \end{array}$$

Proofs with Quantifiers

Suppose we know that $\exists x S(x)$, that we introduce c as the name of an object, and that we assume $S(c)$. If we can conclude a sentence Q not involving c , then we can conclude that Q follows from the original premises.

$$\begin{array}{|l}
 \exists x S(x) \\
 \boxed{c} S(c) \\
 \dots \\
 Q \\
 \hline
 Q \quad \exists \text{Elim}
 \end{array}
 \qquad
 \begin{array}{|l}
 \forall x (\neg \text{Tet}(x) \vee \text{Cube}(b)) \\
 \exists x \text{Tet}(x) \\
 \hline
 \text{Cube}(b)
 \end{array}$$

Existential Elimination
Existential Instantiation

Proofs with Quantifiers

Suppose we know that $\exists x S(x)$, that we introduce c as the name of an object, and that we assume $S(c)$. If we can conclude a sentence Q not involving c , then we can conclude that Q follows from the original premises.

$$\begin{array}{|l}
 \exists x S(x) \\
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 \dots \\
 Q \\
 \hline
 Q \quad \exists \text{Elim}
 \end{array}
 \qquad
 \begin{array}{|l}
 \forall x (\neg \text{Tet}(x) \vee \text{Cube}(b)) \\
 \exists x \text{Tet}(x) \\
 \boxed{c} \text{Tet}(c) \\
 \hline
 \neg \text{Tet}(c) \vee \text{Cube}(b) \\
 \text{Cube}(b)
 \end{array}$$

Existential Elimination
Existential Instantiation

Proofs with Quantifiers

Suppose we know that $\exists x S(x)$, that we introduce c as the name of an object, and that we assume $S(c)$. If we can conclude a sentence Q not involving c , then we can conclude that Q follows from the original premises.

$$\begin{array}{|l}
 \exists x S(x) \\
 \boxed{c} S(c) \\
 \dots \\
 Q \\
 \hline
 Q \quad \exists \text{Elim}
 \end{array}
 \qquad
 \begin{array}{|l}
 \forall x (\neg \text{Tet}(x) \vee \text{Cube}(b)) \\
 \exists x \text{Tet}(x) \\
 \boxed{c} \text{Tet}(c) \\
 \hline
 \neg \text{Tet}(c) \vee \text{Cube}(b) \\
 \text{Cube}(b)
 \end{array}$$

Existential Elimination
Existential Instantiation

Proofs with Quantifiers

Suppose we know that $\exists x S(x)$, that we introduce c as the name of an object, and that we assume $S(c)$. If we can conclude a sentence Q not involving c , then we can conclude that Q follows from the original premises.

$\begin{array}{l} \exists x S(x) \\ \boxed{c} S(c) \\ \dots \\ Q \\ \hline Q \end{array}$ <p style="text-align: right; color: blue;">∃ Elim</p>	$\begin{array}{l} \forall x (\neg \text{Tet}(x) \vee \text{Cube}(b)) \\ \boxed{\exists x \text{Tet}(x)} \\ \boxed{c} \text{Tet}(c) \\ \neg \text{Tet}(c) \vee \text{Cube}(b) \\ \text{Cube}(b) \\ \hline \text{Cube}(b) \end{array}$
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Existential Elimination
Existential Instantiation

Existential Elimination

$\begin{array}{l} \exists x S(x) \\ \boxed{c} S(c) \\ \dots \\ Q \\ \hline Q \end{array}$	$\begin{array}{l} \dots \\ \exists x P(x) \\ \boxed{a} P(a) \\ \dots \\ \exists \dots \\ \hline \exists \dots \quad \exists \text{ Elim} \end{array}$
---	---

Existential Elimination

$\begin{array}{l} \exists x S(x) \\ \boxed{c} S(c) \\ \dots \\ Q \\ \hline Q \end{array}$	$\begin{array}{l} \dots \\ \exists x P(x) \\ \boxed{a} P(a) \\ \dots \\ \exists \dots \\ \hline \exists \dots \quad \exists \text{ Elim} \end{array}$ <p style="color: blue; font-size: small;">Does not involve the constant a</p>
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13.12

$\begin{array}{l} \forall x (\text{Cube}(x) \vee \text{Tet}(x)) \\ \exists x \neg \text{Cube}(x) \\ \hline \end{array}$	$\exists x \text{Tet}(x)$
---	---------------------------

13.12

$\begin{array}{l} \forall x (\text{Cube}(x) \vee \text{Tet}(x)) \\ \exists x \neg \text{Cube}(x) \\ \hline \boxed{a} \neg \text{Cube}(a) \\ \hline \exists x \text{Tet}(x) \quad \exists \text{ Intro} \\ \hline \exists x \text{Tet}(x) \quad \exists \text{ Elim} \end{array}$	
--	--

13.12

<ol style="list-style-type: none"> 1. $\forall x (\text{Cube}(x) \vee \text{Tet}(x))$ 2. $\exists x \neg \text{Cube}(x)$ 3. $\boxed{a} \neg \text{Cube}(a)$ 4. $\text{Cube}(a) \vee \text{Tet}(a)$ 5. $\text{Tet}(a)$ 6. $\exists x \text{Tet}(x)$ 7. $\exists x \text{Tet}(x)$ 	<p style="color: green; font-size: small;">∃ Elim 1 Taut Con 3, 4 ∃ Intro 5 ∃ Elim 2, 3 - 6</p>
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12.23

Proofs with Quantifiers

Every child is right-handed or intelligent
 No intelligent child eats liver
 There is a child who eats liver and onions

There is a right-handed child who eats onions

12.23

1 $\forall x (R(x) \vee I(x))$;All right-handed or intelligent
 2 $\forall x (I(x) \rightarrow \neg L(x))$;No intelligent child eats liver
 3 $\exists x (L(x) \wedge O(x))$;Some child eats liver and onions

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13
14 $\exists x (R(x) \wedge O(x))$;Some right-handed child eats onions

12.23

1 $\forall x (R(x) \vee I(x))$;All right-handed or intelligent
 2 $\forall x (I(x) \rightarrow \neg L(x))$;No intelligent child eats liver
 3 $\exists x (L(x) \wedge O(x))$;Some child eats liver and onions

4 $\boxed{a} \forall L(a) \wedge O(a)$

5
6
7
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11
12
13 $\exists x (R(x) \wedge O(x))$ ∃ Intro
 14 $\exists x (R(x) \wedge O(x))$ ∃ Elim 4-13, 3

12.23

1 $\forall x (R(x) \vee I(x))$;All right-handed or intelligent
 2 $\forall x (I(x) \rightarrow \neg L(x))$;No intelligent child eats liver
 3 $\exists x (L(x) \wedge O(x))$;Some child eats liver and onions

4 $\boxed{a} \forall L(a) \wedge O(a)$

5 $L(a)$ ∧ Elim 4
 6 $O(a)$ ∧ Elim 4

7
8
9
10
11
12
13 $\exists x (R(x) \wedge O(x))$ ∃ Intro
 14 $\exists x (R(x) \wedge O(x))$ ∃ Elim 4-13, 3

12.23

1 $\forall x (R(x) \vee I(x))$;All right-handed or intelligent
 2 $\forall x (I(x) \rightarrow \neg L(x))$;No intelligent child eats liver
 3 $\exists x (L(x) \wedge O(x))$;Some child eats liver and onions

4 $\boxed{a} \forall L(a) \wedge O(a)$

5 $L(a)$ ∧ Elim 4
 6 $O(a)$ ∧ Elim 4
 7 $I(a) \rightarrow \neg L(a)$ ∨ Elim 2
 8 $L(a) \rightarrow \neg I(a)$ Taut Con 7

9
10
11
12
13 $\exists x (R(x) \wedge O(x))$ ∃ Intro
 14 $\exists x (R(x) \wedge O(x))$ ∃ Elim 4-13, 3

12.23

1 $\forall x (R(x) \vee I(x))$;All right-handed or intelligent
 2 $\forall x (I(x) \rightarrow \neg L(x))$;No intelligent child eats liver
 3 $\exists x (L(x) \wedge O(x))$;Some child eats liver and onions

4 $\boxed{a} \forall L(a) \wedge O(a)$

5 $L(a)$ ∧ Elim 4
 6 $O(a)$ ∧ Elim 4
 7 $I(a) \rightarrow \neg L(a)$ ∨ Elim 2
 8 $L(a) \rightarrow \neg I(a)$ Taut Con 7
 9 $\neg I(a)$ → Elim 5, 8

10
11
12
13 $\exists x (R(x) \wedge O(x))$ ∃ Intro
 14 $\exists x (R(x) \wedge O(x))$ ∃ Elim 4-13, 3

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