CS103A

10/15/08

Review Session

Monday or Tuesday
Time: TBA
Location: TBA

Midterm Exam

Thurs., Oct. 23
7 - 9 pm
Location: TBA
Open book (LPL),
Open Notes, Crib Sheet
Coverage: through 10/20

Lewis Carroll Examples

H(x)	x is a hummingbird
RC(x)	x is richly colored (⇔ ¬Dull(x))
LOH(x)	x lives on honey
L(x)	x is large (⇔ ¬Small(x))

Universe of discourse: Birds

All hummingbirds are richly colored.

$\forall x\,(H(x) \rightarrow RC(x))$

No large birds live on honey.

Lewis Carroll Examples

Birds that do not live on honey are dull in color.
Birds that do not live on honey are not richly colored.

Lewis Carroll Examples

Hummingbirds are small.

Lewis Carroll Examples

All hummingbirds are richly colored.
No large birds live on honey.
Birds that do not live on honey are not richly colored.
Hummingbirds are not large.

Lewis Carroll Examples

All hummingbirds are richly colored.
No large birds live on honey.
Birds that do not live on honey are not richly colored.
Hummingbirds are not large.

Large birds are not richly colored.

Since hummingbirds are richly colored,
hummingbirds are not large.

Multiple Quantifiers

$\exists x \exists y\ (Boy(x) \land Girl(y) \land Likes(x, y))$

$\forall x \forall y\ [(Tet(x) \land Dodec(y)) \rightarrow Smaller(x, y)]$

Multiple Quantifiers

$\exists x \exists y\ (Boy(x) \land Girl(y) \land Likes(x, y))$

$\forall x \forall y\ [(Tet(x) \land Dodec(y)) \rightarrow Smaller(x, y)]$

$\forall x \forall y\ ((Cube(x) \land Cube(y)) \rightarrow \ldots)$

Could x and y be the same cube?

Multiple Quantifiers

$\exists x \exists y\ (Boy(x) \land Girl(y) \land Likes(x, y))$

$\forall x \forall y\ [(Tet(x) \land Dodec(y)) \rightarrow Smaller(x, y)]$

$\forall x \forall y\ ((Cube(x) \land Cube(y) \land x \neq y) \rightarrow \ldots)$

$\exists x \exists y\ (Cube(x) \land Cube(y) \land x \neq y)$

Universe of Discourse

There is a universe for each predicate:

Cube	blocks
Boy, Girl, Likes	people
H	birds

When we use quantifiers with predicates, we mean that the quantifiers are applied over the appropriate domain.

Universe of Discourse

HasTaken(a, b) — a has taken class b

If we write $\forall x\ \forall y$ HasTaken (x, y), it is understood that the quantifiers operate over the appropriate domains.

$\forall x\ \forall y$ HasTaken (x, y)

$\exists x\ \exists y$ HasTaken (x, y)

$\exists x\ \forall y$ HasTaken (x, y)

$\forall x\ \exists y$ HasTaken (x, y)

$\exists y\ \forall x$ HasTaken (x, y)

$\forall y\ \exists x$ HasTaken (x, y)

Q(x,y) : x + y = 0

∃y ∀x Q(x, y)

∀x ∃y Q(x, y)

Summary of Quantifiers in Arity-Two Predicates

Statement	When true?	When false?
∀x ∀y P(x,y)	P(x,y) is true for every pair (x,y)	A pair (x,y) exists where P(x,y) is false
∀x ∃y P(x,y)	For every x, there is a y for which P(x,y) is true	There is an x such that P(x,y) is false for every y
∃x ∀y P(x,y)	There is an x for which P(x,y) is true for every y	For every x, there is a y for which P(x,y) is false
∃x ∃y P(x,y)	There is a pair (x,y) for which P(x,y) is true	P(x,y) is false for all pairs (x,y)
∀y ∃x P(x,y)		
∃y ∀x P(x,y)		

What if we wanted to test alternatives to determine if ∀x∀y P(x, y) is true?

```
foreach (x)
     foreach (y)
          if ¬P(x,y) then return F
return T
```

∀x∃y P(x, y)

```
foreach (x)
     foreach (y)
          if P(x,y) then next x
     return F
```

∀x∃y P(x, y)

```
foreach (x)
     foreach (y)
          if P(x,y) then next x
     return F
return T
```

∃x ∀y P(x, y)

```
foreach (x)
     foreach (y)
          if ¬P(x,y) then next x
     return T
return F
```

$\exists x \exists y P(x, y)$

```
foreach (x)
     foreach (y)
          if P(x,y) then return T
return F
```

$Q(x, y, z)$: $x + y = z$

$\forall x \forall y \exists z Q(x, y, z)$

$\exists z \forall x \forall y Q(x, y, z)$

$L(x, y)$: x loves y

1. Everybody loves Fred.
2. Everybody loves somebody.
3. The is somebody whom nobody loves.
4. Everybody loves himself or herself.

11.16

1) Every cube is to the left of every tetrahedron.

2) Every small cube is in back of a large cube.

3) Some cube is in front of every tetrahedron.

4) A large cube is in front of a small cube.

5) Nothing is larger than everything.

6) Every cube in front of every tetrahedron is large.

7) Everything to the right of a large cube is small.

8) Nothing in back of a cube and in front of a cube is large.

9) Anything with nothing in back of it is a cube.

10) Every dodecahedron is smaller than some tetrahedron.