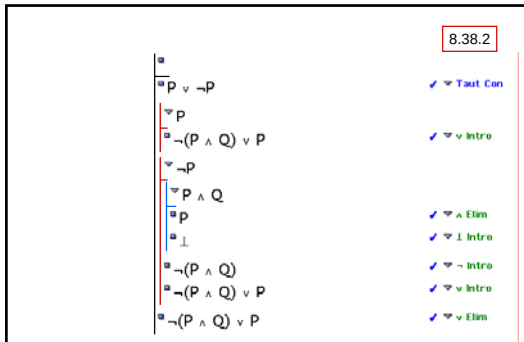
CS103A

10/13/08

Midterm Exam

Thurs., Oct. 23
7 - 9 pm
Location TBA



Describe a world for each of these

∀x Cube(x)

∃x Cube(x)

¬∀x Cube(x)

¬∃x Cube(x)

∀x ¬Cube(x)

∃x ¬Cube(x)

∃x (Professor(x) → Smart(x))
∃x Professor(x)

∃x Smart(x)

∀x (Professor(x) → Smart(x))
∀x Professor(x)

∀x Smart(x)

Tautologies with Quantifiers

P ∨ ¬P

(∀x Cube(x)) ∨ (∀x ¬Cube(x))

(∀x Cube(x)) ∨ (¬∀x Cube(x))

Truth-Functional Form

A way to determine whether a quantified sentence is a tautology.

-- Replace quantified constituents with a letter
-- Replace remaining atomic constituents with a letter
-- Reuse the same letter for identical constituents
-- Maintain connectives outside of quantifiers
-- Check the result

[∀z (Cube(z) → Large(z)) ∧ Cube(b)] → Large(b)

[(∀x Cube(x) → ∀x Small(x)) ∧ ¬∀x Small(x)] → ¬∀x Cube(x)

Cube(a) ∧ Cube(b)

Small(a) ∧ Large(b)

∃x(Cube(x) ∧ Small(x)) ∧ ∃x(Cube(x) ∧ Large(x))

∀x Cube(x) → ∃y Small(y)

¬∃y Small(y)

¬ ∀x Cube(x)

Propositional Logic	*First-Order Logic*	*General Notion*
Tautology	FO Validity	Logical Truth
Tautological Consequence	FO Consequence	Logical Consequence
Tautological Equivalence	FO Equivalence	Logical Equivalence

Connectives

Connectives
Identity
Quantifiers

Connectives
Identity
Quantifiers
Meanings of predicates and function symbols

Cube(a) → ∃x Cube(x)

FO Consequence?

Cube(a) ∧ Cube(b)

Small(a) ∧ Large(b)

∃x(Cube(x) ∧ Large(x) ∧ ¬Smaller(x, x))

FO Consequence?

P(a) ∧ P(b)

Q(a) ∧ R(b)

∃x(P(x) ∧ R(x) ∧ ¬S (x, x))

Counterexample:

P(x) : x is a number
a: 1
b: 2
Q(x): x is odd
R(x): x is even
S(x,y): x = y

FO Consequence?

Cube(a) ∧ Cube(b)

Small(a) ∧ Large(b)

∃x(Cube(x) ∧ Large(x) ∧ ¬Smaller(x, x))

FO Con 1

1. ∃x Dodec(x) → ∃y Small(y)
2. ¬∃y Small(y)
3. ∀x (Cube(x) → Large(x))

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4. Medium(c)
5. ∀x Cube(x) → Cube(b)
6. ¬∃y Dodec(y) 1,2
7. ¬∃x Dodec(x) 1,2
8. ∀x (¬Large(x) → ¬Cube(x)) 3
9. ¬∃y Small(y) → ¬∃x Dodec(x) 1
10. ¬Cube(c) 3,4