

Proof Strategies

Where To Start?

- As you begin a proof, convince yourself that the conclusion is indeed true by studying the premises and understanding their meaning:
- Analyze the structure of each premise and the conclusion.
- Make sure you understand all the terms and operations used in the premises and conclusion—get your definitions ready. Replace terms with their definitions to get clues on how to proceed. (This will be important when we are doing math proofs.)
- We typically prove general conclusions about a set of objects. One way to understand what is to be proved is by proving the conclusion for a particular element of the set. Convince yourself that the conclusion is true for this element (and if you find that it is not, you have just disproved the conclusion). This will often give you insight into how to do the general proof. For the kinds of proofs we are doing now, examples would be creating a particular Tarski's World and asking why the general principle holds there, or replacing P's, Q's, etc. in an abstract proof with Tarski's World predicates and then trying the proof. This is one of the most important techniques for mathematicians to gain insight: **Try special cases** and generalize from those cases.
- Try giving an informal proof of the conclusion—explain it to a friend.

A Common Methodology: Working Backwards

Once you have convinced yourself that the conclusion is valid, there are two possible ways to proceed:

1. If the process you went through in understanding the premises and conclusion (working through an example or defining an informal proof) gave you all the details you need, write up a formal (or informal depending on the requirements) proof following the same sequence of steps. In writing an informal proof, you may leave out obvious steps if you know your audience can fill in those gaps.
2. If you are convinced of the conclusion's validity but are having trouble formalizing it, try working backwards from the conclusion, rather than forward from the premises. That is, ask yourself what intermediate step would allow you to prove the conclusion,

and then try to prove that intermediate step on the basis of the available information. Think carefully about each step, whether working forwards or backwards.

How To End

Check your work: Remember that each line in a proof (whether formal or informal) is either a premise, an intermediate step or the final goal. Everything other than the premises must be justified. Make certain that the justifications are valid and lead step-by-step to the conclusion.

If you are writing a formal proof, the justifications must be explicitly stated. If you are writing an informal proof, certain obvious justifications can be left out assuming that your audience can still follow the proof.

A well-written proof flows. The reader should be led directly and inevitably to the desired conclusion without any distractions or irrelevant details. In an informal proof, clarifying remarks are sometimes helpful.

(Original handout by Maggie Johnson, modified by Robert Plummer)