

Proof of Resolution Principle
 (Proof 6.19)

$A \vee B$	
$\neg B \vee C$	
A	
$A \vee C$	\vee Intro
B	
$\neg B$	
\perp	\perp Intro
$A \vee C$	\perp Elim
C	
$A \vee C$	\vee Intro
$A \vee C$	\vee Elim
$A \vee C$	\vee Elim

Section 6.4 (page 166)

6.19

$A \vee B$	
$\neg B \vee C$	
A	
$\neg B$	
$A \vee C$	Rule?
C	
$A \vee C$	Rule?
$A \vee C$	\vee Elim
B	
$\neg B$	
\perp	Rule?
$A \vee C$	Rule?
C	
$A \vee C$	Rule?
$A \vee C$	\vee Elim
$A \vee C$	\vee Elim

Two Ways To Use Contradictions

Proof by Contradiction (\neg -Intro) Negation Elimination (\neg -Elim)

$\neg P$	P		
.	.		
.	.		
.	.		
\perp	\perp		
P	$\neg P$		
			Q
			(for any Q)

This is usually, as shown, in a subproof, and we choose Q to be the conclusion of the subproof.

6.9

$Cube(b)$	
$\neg(Cube(c) \wedge Cube(b))$	
$\neg Cube(c)$	

6.9

$Cube(b)$	
$\neg(Cube(c) \wedge Cube(b))$	
$Cube(c)$	
\perp	
$\neg Cube(c)$	\neg Intro

6.9

$Cube(b)$	
$\neg(Cube(c) \wedge Cube(b))$	
$Cube(c)$	
$Cube(c) \wedge Cube(b)$	\wedge Intro
\perp	\perp Intro
$\neg Cube(c)$	\neg Intro

Negation 2 in Fitch

Negation 4

$\neg(\neg\text{Dodec}(b) \vee \neg\text{Dodec}(c))$	
$\text{Dodec}(b) \wedge \text{Dodec}(c)$	

6.28

$\text{Cube}(c) \vee \text{Small}(c)$ $\text{Dodec}(c)$	
$\text{Small}(c)$	

6.34

$\neg(a = b \wedge \text{Dodec}(a) \wedge \neg\text{Dodec}(b))$	
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6.34

$a = b \wedge \text{Dodec}(a) \wedge \neg\text{Dodec}(b)$	
\perp	
$\neg(a = b \wedge \text{Dodec}(a) \wedge \neg\text{Dodec}(b))$	\neg Intro

6.34

$a = b \wedge \text{Dodec}(a) \wedge \neg\text{Dodec}(b)$	
$a = b$	\wedge Elim
$\text{Dodec}(a)$	\wedge Elim
$\neg\text{Dodec}(b)$	\wedge Elim
\perp	
$\neg(a = b \wedge \text{Dodec}(a) \wedge \neg\text{Dodec}(b))$	\neg Intro

6.34

$a = b \wedge \text{Dodec}(a) \wedge \neg \text{Dodec}(b)$	
$a = b$	\wedge Elim
$\text{Dodec}(a)$	\wedge Elim
$\neg \text{Dodec}(b)$	\wedge Elim
$\text{Dodec}(b)$	$=$ Elim
\perp	\perp Intro
$\neg (a = b \wedge \text{Dodec}(a) \wedge \neg \text{Dodec}(b))$	\neg Intro

6.29

$\text{Larger}(a, b) \vee \text{Larger}(a, c)$	
$\text{Smaller}(b, a) \vee \neg \text{Larger}(a, c)$	
$\text{Larger}(a, b)$	

Rapid-fire Examples

- 1) Given: $(A \vee B) \wedge (A \vee C)$
 Prove: A
- 2) Given: $(A \vee B) \wedge (A \wedge C)$
 Prove: A
- 3) Given: $(A \wedge B) \vee (A \wedge C)$
 Prove: $A \wedge (B \vee C)$
- 4) Given: $A \vee B$
 $C \vee D$
 $\neg C \wedge \neg B$
 Prove: $A \wedge D$
- 5) Given: $(A \wedge B) \wedge C$
 $\neg A$
 Prove: C