Rules of Inference in Fitch

Conjunction Elimination ($\wedge$ Elim)

$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$	
P_i	$\wedge$ Elim

Conjunction Introduction ($\wedge$ Intro)

P_1	
P_2	
$\Downarrow$	
P_n	
. . .	
$P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n$	$\wedge$ Intro

Proof 6.3

1. $a = b \wedge b = c \wedge c = d$

Goal: $a = c \wedge b = d$

Proof 6.3

1. $a = b \wedge b = c \wedge c = d$	
2. $a = b$	$\wedge$ Elim: 1
3. $b = c$	$\wedge$ Elim: 1
4. $a = c$	= Elim: 2, 3

Goal: $a = c \wedge b = d$

Proof 6.3

1. $a = b \wedge b = c \wedge c = d$	
2. $a = b$	$\wedge$ Elim: 1
3. $b = c$	$\wedge$ Elim: 1
4. $a = c$	= Elim: 2, 3
5. $c = d$	$\wedge$ Elim: 1
6. $b = d$	= Elim: 3, 5

Goal: $a = c \wedge b = d$

Proof 6.3

1. a = b ∧ b = c ∧ c = d
2. a = b ∧ Elim: 1
3. b = c ∧ Elim: 1
4. a = c = Elim: 2, 3
5. c = d ∧ Elim: 1
6. b = d = Elim: 3, 5
7. a = c ∧ b = d ∧ Intro: 4, 6

Goal: a = c ∧ b = d

Disjunction Introduction (∨ Intro)

P
.
.
.
P ∨ Q ∨ Intro

Disjunction Elimination (∨ Elim)
Proof by Cases

$P_1 \vee \dots \vee P_n$

P_1
...
S

⇓

P_n
...
S

S ∨ Elim

Proof 6.4

(A ∧ B) ∨ C

Goal: C ∨ B

Proof 6.4

(A ∧ B) ∨ C
A ∧ B

C

Goal: C ∨ B

Proof 6.4

(A ∧ B) ∨ C
A ∧ B
C ∨ B
C
C ∨ B

Goal: C ∨ B

Proof 6.4

(A ∧ B) ∨ C
A ∧ B
C ∨ B
C
C ∨ B
C ∨ B ∨ Elim

Goal: C ∨ B

Proof 6.4

(A ∧ B) ∨ C
A ∧ B
C ∨ B
C
C ∨ B
C ∨ B ✓ ∨ Elim

Goal: C ∨ B

Proof 6.4

(A ∧ B) ∨ C
A ∧ B
B ∧ Elim
C ∨ B ∨ Intro
C
C ∨ B ∨ Intro
C ∨ B ∨ Elim

Goal: C ∨ B

Proof 6.4

1. (A ∧ B) ∨ C
2. A ∧ B
3. B ✓ ∧ Elim: 2
4. C ∨ B ✓ ∨ Intro: 3
5. C
6. C ∨ B ✓ ∨ Intro: 5
7. C ∨ B ✓ ∨ Elim: 1,2-4,5-6

Goal: C ∨ B ✓

Negation Elimination (¬ Elim)

¬¬P
.
.
.
P ¬ Elim

Proof by Contradiction
Indirect Proof

Suppose that from premises P_1, …, P_n we wish to show S.

One possibility is to temporarily assume ¬S, and see if we can show that P_1, …, P_n , ¬S is impossible, i.e., that these claims can't be true simultaneously.

Then in any circumstances where P_1, …, P_n are true,

¬S is false

Proof by Contradiction
Indirect Proof

Suppose that from premises $P_1, \ldots, P_n$ we wish to show S.

One possibility is to temporarily assume ¬S, and see if we can show that $P_1, \ldots, P_n$, ¬S is impossible, i.e., that these claims can't be true simultaneously.

Then in any circumstances where $P_1, \ldots, P_n$ are true,

¬S is false
¬ ¬ S is true

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Indirect Proof

Suppose that from premises $P_1, \ldots, P_n$ we wish to show S.

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Then in any circumstances where $P_1, \ldots, P_n$ are true,

¬S is false
¬ ¬ S is true
S is true

Proof by Contradiction
Indirect Proof

Suppose that from premises $P_1, \ldots, P_n$ we wish to show S.

One possibility is to temporarily assume ¬S, and see if we can show that $P_1, \ldots, P_n$, ¬S is impossible, i.e., that these claims can't be true simultaneously.

Then in any circumstances where $P_1, \ldots, P_n$ are true,

¬S is false
¬ ¬ S is true
S is true

So S follows from $P_1, \ldots, P_n$

Proof by Contradiction
Indirect Proof

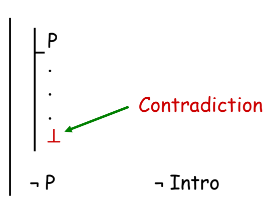
Suppose that from premises $P_1, \ldots, P_n$ we wish to show ¬S.

One possibility is to temporarily assume S, and see if we can show that $P_1, \ldots, P_n$, S is impossible, i.e., that these claims can't be true simultaneously.

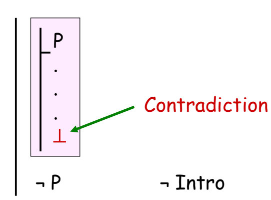
Then in any circumstances where $P_1, \ldots, P_n$ are true,

S is false
¬ S is true

Negation Introduction (¬ Intro)
Proof by Contradiction
Indirect Proof



Negation Introduction (¬ Intro)
Proof by Contradiction
Indirect Proof



⊥ Introduction (⊥ Intro)

| P |
| . |
| . |
| . |
| ¬ P |
| . |
| . |
| . |
| ⊥ ⊥ Intro |

Negation Introduction (¬ Intro)
Proof by Contradiction
Indirect Proof

	P
	.
	.
	.
	⊥ ⊥ Intro
¬ P ¬ Intro

Negation Introduction (¬ Intro)
Proof by Contradiction
Indirect Proof

1. | Q
2. | | P
3. | | .
4. | | .
5. | | ¬Q
6. | | ⊥ ⊥ Intro: 1, 5
7. | ¬ P ¬ Intro: 2-6

⊥ Elimination (⊥ Elim)

| ⊥ |
| . |
| . |
| . |
| P ⊥ Elim |

This can be any sentence

We generally apply this rule in a subproof.

How to show a contradiction

Prove something and its negation, e.g., Q and ¬Q

Prove sentences that can't simultaneously be true in the blocks world and use AnaCon, e.g., Larger(a, b) and Larger(b, a)

Given: n^2 is odd
Prove: n is odd

PROOF: Suppose n is even.

Then n = 2k for some integer k, and

$n^2 = 4k^2$

But $4k^2$ is divisible by 2, which means that n^2 is even, contradicting the premise that n^2 is odd.

Thus, if n^2 is odd, then n is odd.

Proof 6.13

Dodec(e)
Large(e)
¬Dodec(e) ∨ Dodec(f) ∨ Small(e)

Dodec(f)

Proof 6.13

Dodec(e)
Large(e)
¬Dodec(e) ∨ Dodec(f) ∨ Small(e)

¬ Dodec(e)

Dodec(f)

Dodec(f)
Dodec(f)

Small(e)

Dodec(f)

Dodec(f) ∨ Elim

Proof 6.13

Dodec(e)
Large(e)
¬Dodec(e) ∨ Dodec(f) ∨ Small(e)

¬ Dodec(e)
⊥ ⊥ Intro
Dodec(f) ⊥ Elim

Dodec(f)
Dodec(f) Reit

Small(e)
⊥ ?????
Dodec(f) ⊥ Elim

Dodec(f) ∨ Elim

Proof 6.13

Dodec(e)
Large(e)
¬Dodec(e) ∨ Dodec(f) ∨ Small(e)

¬ Dodec(e)
⊥ ⊥ Intro
Dodec(f) ⊥ Elim

Dodec(f)
Dodec(f) Reit

Small(e)
⊥ Ana Con
Dodec(f) ⊥ Elim

Dodec(f) ∨ Elim

Example

(¬P ∨ R) ∧ (¬Q ∨ R)

¬(P ∨ Q) ∨ R

Example

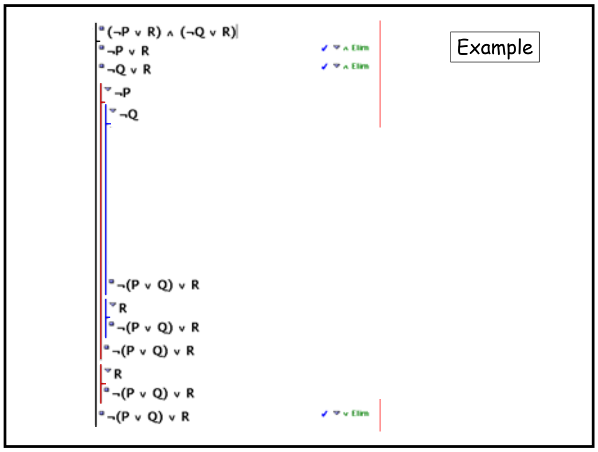
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim

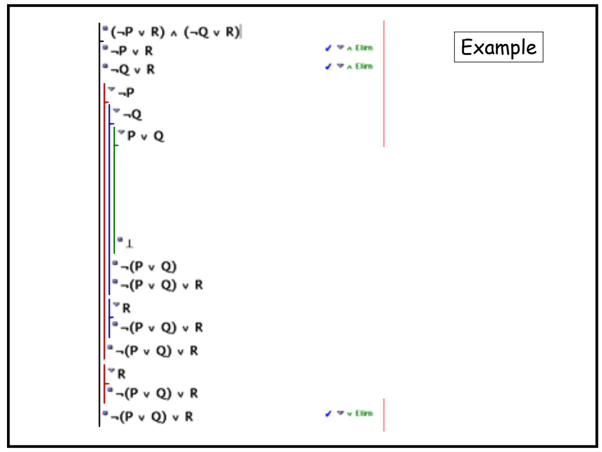
¬P

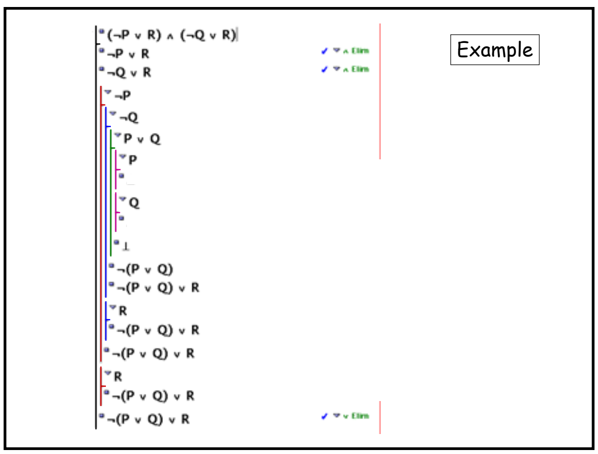
¬(P ∨ Q) ∨ R

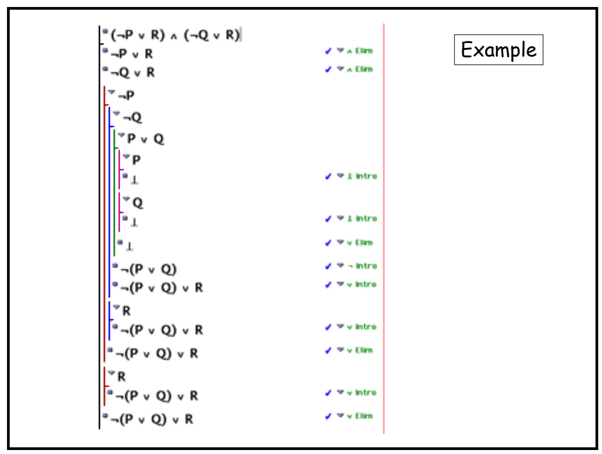
R
¬(P ∨ Q) ∨ R

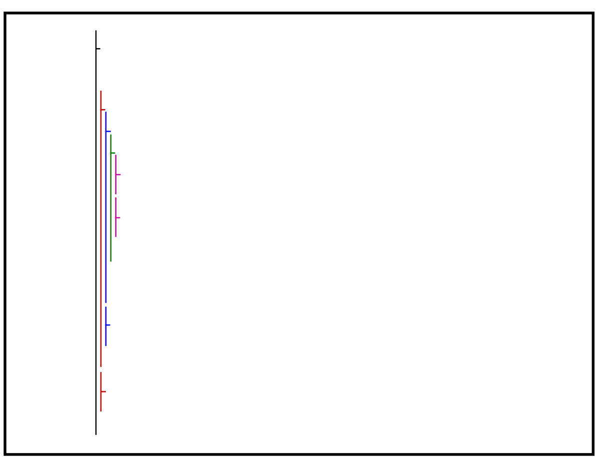
¬(P ∨ Q) ∨ R ∨ Elim

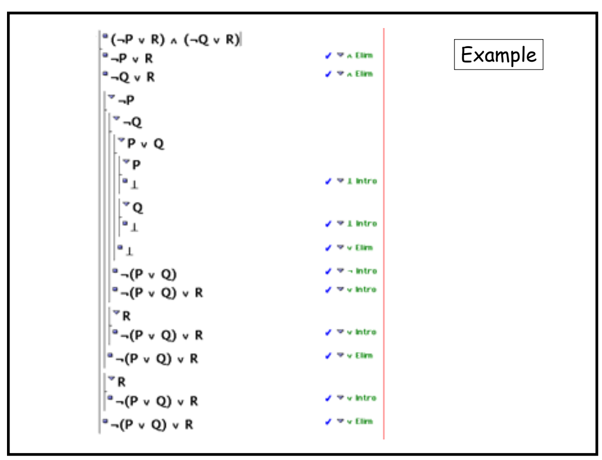

Example
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim
¬P
¬Q
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R ∨ Elim


Example
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim
¬P
¬Q
P ∨ Q
⊥
¬(P ∨ Q)
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R ∨ Elim


Example
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim
¬P
¬Q
P ∨ Q
P
Q
⊥
¬(P ∨ Q)
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R
R
¬(P ∨ Q) ∨ R
¬(P ∨ Q) ∨ R ∨ Elim


Example
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim
¬P
¬Q
P ∨ Q
P
⊥ ⊥ Intro
Q
⊥ ⊥ Intro
⊥ ∨ Elim
¬(P ∨ Q) ¬ Intro
¬(P ∨ Q) ∨ R ∨ Intro
R
¬(P ∨ Q) ∨ R ∨ Intro
¬(P ∨ Q) ∨ R ∨ Elim
R
¬(P ∨ Q) ∨ R ∨ Intro
¬(P ∨ Q) ∨ R ∨ Elim




Example
(¬P ∨ R) ∧ (¬Q ∨ R)
¬P ∨ R ∧ Elim
¬Q ∨ R ∧ Elim
¬P
¬Q
P ∨ Q
P
⊥ ⊥ Intro
Q
⊥ ⊥ Intro
⊥ ∨ Elim
¬(P ∨ Q) ¬ Intro
¬(P ∨ Q) ∨ R ∨ Intro
R
¬(P ∨ Q) ∨ R ∨ Intro
¬(P ∨ Q) ∨ R ∨ Elim
R
¬(P ∨ Q) ∨ R ∨ Intro
¬(P ∨ Q) ∨ R ∨ Elim

Proof of Resolution Principle

```
| A ∨ ¬B
| B ∨ C
|---
|
| A ∨ C
```

Proof of Resolution Principle

```
| A ∨ ¬B
| B ∨ C
|---
| | A
| |---
| | A ∨ C
| | ¬B
| |---
| |
| | A ∨ C
| A ∨ C                      ∨ Elim
```

Proof of Resolution Principle

```
| A ∨ ¬B
| B ∨ C
|---
| | A
| |---
| | A ∨ C
| | ¬B
| |---
| |
| | A ∨ C
| A ∨ C               ✓ ∨ Elim
```

Proof of Resolution Principle

```
| A ∨ ¬B
| B ∨ C
|---
| | A
| |---
| | A ∨ C                    ∨ Intro
| | ¬B
| |---
| | | B
| | |---
| | | ⊥                      ⊥ Intro
| | | A ∨ C                  ⊥ Elim
| | | C
| | |---
| | | A ∨ C                  ∨ Intro
| | A ∨ C                    ∨ Elim
| A ∨ C                      ∨ Elim
```

Proof of Resolution Principle

```
1.  | A ∨ ¬B
2.  | B ∨ C
    |---
3.  | | A
    | |---
4.  | | A ∨ C          ∨ Intro: 3
5.  | | ¬B
    | |---
6.  | | | B
    | | |---
7.  | | | ⊥            ⊥ Intro: 5, 6
8.  | | | A ∨ C        ⊥ Elim: 7
9.  | | | C
    | | |---
10. | | | A ∨ C        ∨ Intro: 9
11. | | A ∨ C          ∨ Elim: 2, 6-8, 9-10
12. | A ∨ C            ∨ Elim: 1, 3-4, 5-11
```