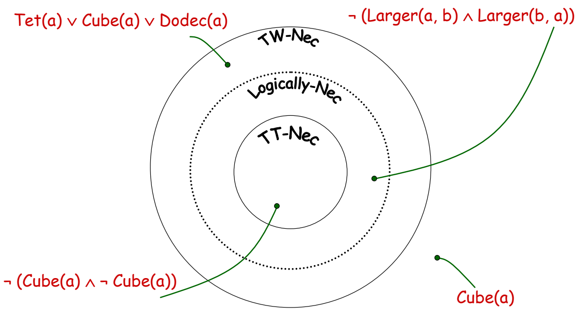
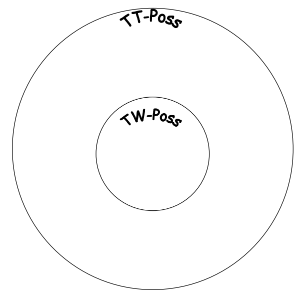
Tautologies / Logical Necessity / Logical Possibility



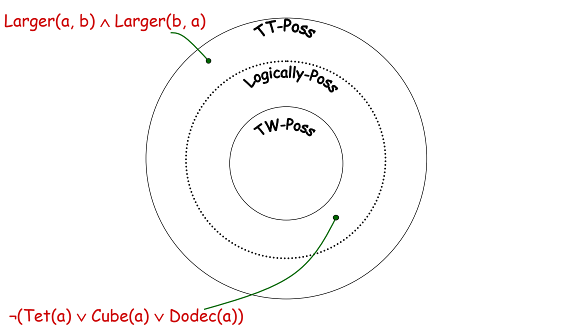
Tautologies / Logical Necessity / Logical Possibility

As we've seen, some things are possible in a truth table that are not possible in Tarski's World.



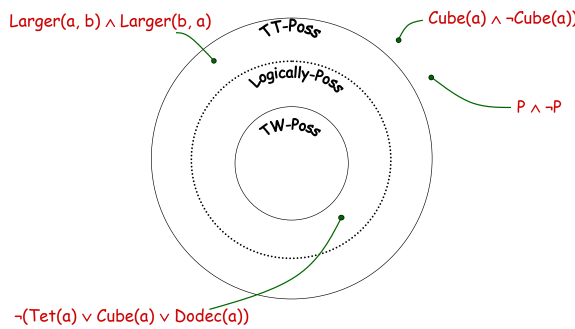
Tautologies / Logical Necessity / Logical Possibility

As we've seen, some things are possible in a truth table that are not possible in Tarski's World.



Tautologies / Logical Necessity / Logical Possibility

As we've seen, some things are possible in a truth table that are not possible in Tarski's World.



Tautologies

Tautologies are sentences that cannot be false, due to their structure and the meanings of the truth-functional connectives they contain.

A	B	C	Main connective ↓
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Tautological Equivalence

We can use test two sentences for tautological equivalence by creating a joint truth table for the sentences and determining whether the values under their main connectives are the same in all rows.

A	B	C	S_1: Main connective ↓	S_2: Main connective ↓
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	F

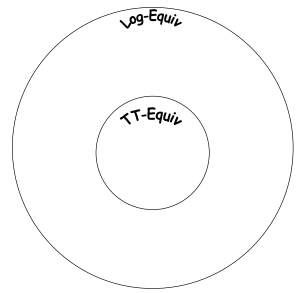
Logical Equivalence

Two sentences that are not tautologically equivalent are considered to be logically equivalent if the only mismatches under their main connectives occur in rows that are impossible if we consider the meanings of the predicates.

L(a,b)	L(b,a)	...	S_1: Main connective	S_2: Main connective
T	T		T	F
T	F		F	F
T	T		F	F
T	F		T	T
F	T		T	T
F	F		F	F
F	T		T	T
F	F		F	F

(where L means Larger)

Equivalence



Important Equivalences Discussed in the Text

Idempotent Laws	$P \vee P \Leftrightarrow P$ $P \wedge P \Leftrightarrow P$
Double Negation	$\neg(\neg P) \Leftrightarrow P$
Commutative Laws	$P \vee Q \Leftrightarrow Q \vee P$ $P \wedge Q \Leftrightarrow Q \wedge P$
Associative Laws	$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$ $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
Distributive Laws	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
DeMorgan's Laws	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

Consequence

The notion that we are really interested in is consequence, since in a proof we are trying to show that a conclusion Q is a consequence of some premises $P_1, P_2, \ldots, P_n$.

Consequence

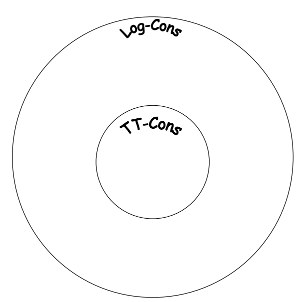
The notion that we are really interested in is consequence, since in a proof we are trying to show that a conclusion Q is a consequence of some premises $P_1, P_2, \ldots, P_n$.

Q is a tautological consequence of $P_1, P_2, \ldots, P_n$ if every row in a joint truth table that assigns T to each of $P_1, P_2, \ldots, P_n$ also assigns T to Q.

Q is a logical consequence of $P_1, P_2, \ldots, P_n$ if Q is true in every possible circumstance where $P_1, P_2, \ldots, P_n$ are true.

....	P_1 P_2 ... P_n	Q

Consequence



Consequence

Be sure you read and understand the Con rules in Fitch:

Taut Con: a theorem prover that ignores the meanings of predicates

FO Con: pays attention to the identity predicate

Ana Con: pays attention to the identity predicate and most blocks world predicates

Consequence

Be sure you read and understand the Con rules in Fitch:

Taut Con: a theorem prover that ignores the meanings of predicates

FO Con: pays attention to the identity predicate

Ana Con: pays attention to the identity predicate and most blocks world predicates

| Larger(b, a)
| Smaller(a, b)

Consequence

Be sure you read and understand the Con rules in Fitch:

Taut Con: a theorem prover that ignores the meanings of predicates

FO Con: pays attention to the identity predicate

Ana Con: pays attention to the identity predicate and most blocks world predicates

| Larger(b, a)
| Smaller(a, b)   Ana Con

Rules of Inference

Conjunction

∧ Intro
| P
| .
| .
| .
| Q
| .
| .
| .
| P ∧ Q

If we know P is true and Q is true, we can infer P ∧ Q

Rules of Inference

Conjunction

∧ Intro
| P
| .
| .
| .
| Q
| .
| .
| .
| P ∧ Q

If we know P is true and Q is true, we can infer P ∧ Q

∧ Elim
| P ∧ Q
| .
| .
| .
| P
| .
| .
| .
| Q

If we know P ∧ Q is true we can infer P and we can infer Q

Rules of Inference

Disjunction

∨ Intro
| P
| .
| .
| .
| P ∨ Q

If we know P is true we can infer P ∨ Q for any Q

Rules of Inference

Disjunction

∨ Intro

| P
| .
| .
| .
| P ∨ Q

If we know P is true we can infer P ∨ Q for any Q

∨ Elim

| P ∨ Q
|
| S

If we know P ∨ Q is true, and S follows from P, and S follows from Q, we can infer S

Rules of Inference

Disjunction

∨ Intro

| P
| .
| .
| .
| P ∨ Q

If we know P is true we can infer P ∨ Q for any Q

∨ Elim

| P ∨ Q
| | P
| | ...
| | S
| | Q
| | ...
| | S
| S

If we know P ∨ Q is true, and S follows from P, and S follows from Q, we can infer S

Rules of Inference

Disjunction

∨ Intro

| P
| .
| .
| .
| P ∨ Q

If we know P is true we can infer P ∨ Q for any Q

∨ Elim

Subproofs

| P ∨ Q
| | P
| | ...
| | S
| | Q
| | ...
| | S
| S

If we know P ∨ Q is true, and S follows from P, and S follows from Q, we can infer S

Proof by Cases

Premise: (P ∧ Q) ∨ (Q ∧ R) Prove: Q

Premise: (P ∧ Q) ∨ (R ∧ S) Prove: P ∨ R

Premises: A ∨ ¬ B
B ∨ C

Prove: A ∨ C

Important proof!