Truth Tables for Boolean Connectives

Negation

P	¬P
T	F
F	T

¬P is true if and only if P is false.

Disjunction

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P ∨ Q is true if and only if P is true, Q is true, or both are true.

Conjunction

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P ∧ Q is true if and only if both P and Q are true.

In the truth table, we are not concerned with what P and Q mean in some world—they stand for two independent propositions that can be true or false.

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To say that the connectives are truth functional means that the value of a sentence using the connective can be determined by the truth value of its constituent parts.

The Henkin-Hintikka Game

Playing the game may help you understand a difficult sentence.

The real significance is that the game adds to you understanding of how the connectives work.

CLAIM	DEMONSTRATION	REFUTATION
P ∧ Q true	Both of P, Q true	One of P, Q false

CLAIM	DEMONSTRATION	REFUTATION
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P ∧ Q false	One of P, Q false	Both of P, Q true
P ∨ Q true	One of P, Q true	Both of P, Q false
P ∨ Q false	Both of P, Q false	One of P, Q true
¬P true	P false	P true
¬P false	P true	P false

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To show that P ∧ Q is false, we show that one of P, Q is false.

¬(P ∧ Q) ¬P ∨ ¬Q

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¬(P ∧ Q) ⇔ ¬P ∨ ¬Q
Similarly ¬(P ∨ Q) ⇔ ¬P ∧ ¬Q } DeMorgan's Laws
and ¬¬ P ⇔ P Double negation

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Similarly ¬(P ∨ Q) ⇔ ¬P ∧ ¬Q
and ¬¬ P ⇔ P

Be SURE that you understand that this is just an abbreviation for "is equivalent to". It is not a Boolean connective.

The statement that two sentences are logically equivalent means that they have the same "truth conditions"; i.e., that they have the same truth value in every possible circumstance.

Tautologies / Logical Necessity / Logical Possibility

To say that a sentence is necessarily true means that it is always true, regardless of the truth or falsity of the atomic sentences it contains. That is, every row of the truth table contains a T under the main connective.

A	B	C	A ∨ ¬ (B ∨ ¬(C ∧ A))

Sentences that are always true are called tautologies.

The truth of a tautology is determined by its structure and the meanings of the connectives. It is independent of the way the world happens to be and the meanings of the constituent atomic sentences.

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T	F	T	
T	F	F	
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T	T	T							T	
T	T	F							F	
T	F	T							T	
T	F	F							F	
F	T	T							F	
F	T	F							F	
F	F	T							F	
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T	T	F						T	F	
T	F	T						F	T	
T	F	F						T	F	
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T	F	F					T	T	F	
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T	T	T			F		T	F	T	
T	T	F			F		T	T	F	
T	F	T			T		F	F	T	
T	F	F			F		T	T	F	
F	T	T			F		T	T	F	
F	T	F			F		T	T	F	
F	F	T			F		T	T	F	
F	F	F			F		T	T	F	

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T	T	T		T	F		T	F	T	
T	T	F		T	F		T	T	F	
T	F	T		T	T		F	F	T	
T	F	F		T	F		T	T	F	
F	T	T		F	F		T	T	F	
F	T	F		F	F		T	T	F	
F	F	T		F	F		T	T	F	
F	F	F		F	F		T	T	F	

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T	T	T		T	F		T	F	T	
T	T	F		T	F		T	T	F	
T	F	T		T	T		F	F	T	
T	F	F		T	F		T	T	F	
F	T	T		F	F		T	T	F	
F	T	F		F	F		T	T	F	
F	F	T		F	F		T	T	F	
F	F	F		F	F		T	T	F	

Not a tautology

TT-possible

Tautologies / Logical Necessity / Logical Possibility

Most sentences are not tautologies. One thing that would be is the negation of something that is always false:

¬ (Cube(a) ∧ ¬ Cube(a)) ⇔ ¬ Cube(a) ∨ Cube(a)

Tautologies / Logical Necessity / Logical Possibility

We might wonder how the notion of tautologies relates to sentences about Tarski's World.

Are the TT-necessary and the TW-necessary sentences the same?

- Anything that is always true is always true in Tarski's World.
- We can have sentences that are TW-nec but not TT-nec. These are sentences that have F in their truth tables only in rows that cannot occur in Tarski's World.

  Example: Tet(a) ∨ Cube(a) ∨ Dodec(a)

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Tet(a)	Cube(a)	Dodec(a)	Tet(a) ∨ Cube(a) ∨ Dodec(a)
F	F	F	F

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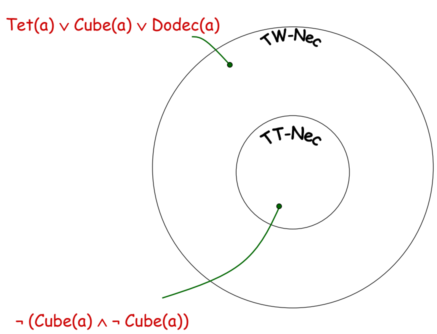
Tet(a)	Cube(a)	Dodec(a)	Tet(a) ∨ Cube(a) ∨ Dodec(a)
F	F	F	F

This row is in the truth table but but it can't occur in Tarski's World World. The truth table does not reflect the meanings of the predicates

Tautologies / Logical Necessity / Logical Possibility



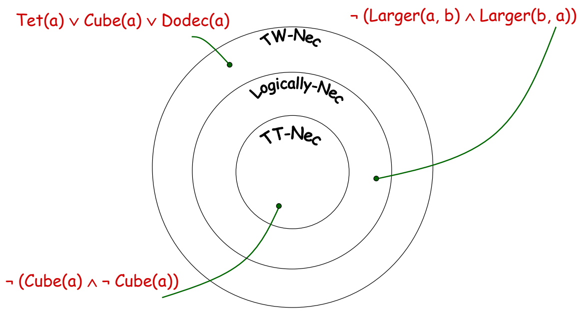
Tautologies / Logical Necessity / Logical Possibility

What about a sentence like this:

¬ (Larger(a, b) ∧ Larger(b, a))

- Some sentences have rows with F under the main connective only in places that are impossible in any blocks world.
- Since all other rows have T, these sentences are said to be logically necessary.
- These sentences are TW-nec, but it is more restrictive to say that the sentence is true in any blocks world than just in the particular Tarski's world.
- The sentence shown is not TT-nec, since it is logically necessary only if we consider the meaning of the predicate Larger.

Tautologies / Logical Necessity / Logical Possibility



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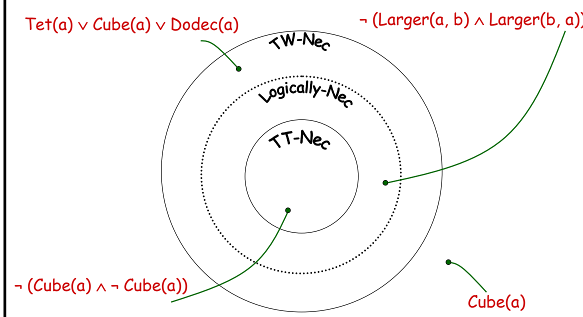

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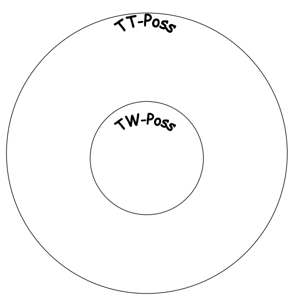
As we've seen, some things are possible in a truth table that are not possible in Tarski's World.



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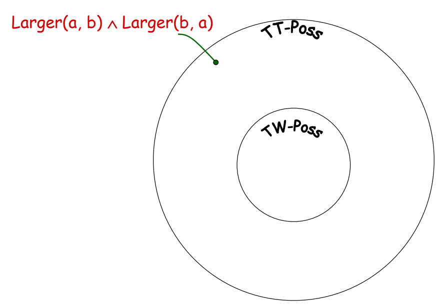

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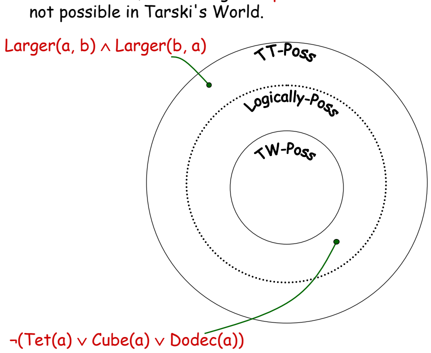

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