

Numerical Methods for Initial Boundary Value Problems (3 units)

Instructor: Bertil Gustafsson

Why should you take this course?

Computer simulation is now a standard tool for almost all problems in science and engineering. It is based on mathematical and numerical models, and the largest class of models is partial differential equations (PDE). In many cases the PDE model is well known, like the Navier-Stokes equations in fluid dynamics and the Maxwell equations in electromagnetics. However, the construction of a numerical model requires thorough knowledge of Numerical Analysis and the fundamental concepts of wellposedness, stability and accuracy. For example, if the result of a certain computation doesn't seem to be accurate enough, one wants to be sure that a new computation on a finer mesh provides a more accurate result. Another typical problem arises if a well known set of PDE is simplified or modified to obtain a faster numerical method. How can one be sure that the new model behaves well and leads to an accurate computation? In both cases, satisfactory behaviour can be guaranteed only if the problem is well posed, and if the numerical method is stable. The question is then how these properties can be analyzed theoretically.

Numerical methods have been around as long as there have been computers. Already in the forties, von Neumann came up with famous von Neumann condition for stability of a difference method. This condition is based on Fourier analysis, and can be applied only for the simplified case with constant coefficients and periodic solutions. Furthermore, it is necessary but not sufficient for stability. Interestingly, after sixty years, it is still in many cases the only tool (if any) that is used for analysis. This is bad, because the construction and application of a numerical method to a given problem, should not be based on guess work and trial and error experiments. For example, how does one handle a case, where the computation shows some suspicious oscillations near the boundary, even if the von Neumann condition is satisfied? The answer is that one has to know the theory for initial-boundary value problems. Rather than trying to eliminate the oscillations by experimenting with different kinds of boundary conditions or smoothing techniques, application of the so called energy method or of the more modern and general normal mode analysis, may quickly solve the problem.

This course provides the basic theory needed for a successful analysis, both for the PDE and for its approximation. The analysis of the PDE itself is necessary to do first, and should be done regardless of the particular numerical method, whether it is based on finite difference, finite volume, finite element or spectral numerical techniques. Among the numerical methods, we limit ourselves in this course to finite difference methods. The strength of these methods is the speed and simplicity of implementation.

The course is to a large extent built on the study of simple model examples for illustration of the basic theory. These examples represent the simplest models for a number of physical phenomena like for example wave propagation, fluid dynamics and heat conduction.

Some knowledge about numerical solution of ordinary differential equations as well as basic linear algebra and matrix computation is a good basis for the course.

If you think that you are going to deal with computer simulations in science or engineering in your future career, then this course is hopefully going to strengthen your ability in that area

considerably. Also, if you are already working with PDE solvers in some application area, this may be a chance to refresh and update your knowledge of the mathematical/numerical theory.

Content: (Preliminary)

1. Well posed problems
2. Periodic problems and Fourier analysis
3. Initial-boundary value problems and the energy method
4. Initial-boundary value problems and normal mode analysis
5. Order of accuracy and convergence rate
6. High order difference methods

Literature:

Bertil Gustafsson: High order difference methods for time-dependent PDE. This is a new book manuscript that is not yet published, and it will be available as a course reader at low cost.

Examination:

There will be 5 mandatory problems to be done as home work. No exam in class.

Time:

Monday and Wednesday, 2.15-3.45 pm
Jan 10 - March 16

Place:

ECON206