Assignment 5

(Due in class March 14)

Consider the differential equation

$$u_t = u_{xx} + F(x,t)\ , \quad 0 \le x \le 2\pi\ ,\ 0 \le t \le 1\ ,$$
$$u(x,0) = f(x)\ ,$$

where $f(x)$ is 2π-periodic.

1. Find the solution for the case

$$f(x) = \sin(\omega x)\ ,$$
$$F(x,t) = (\omega^2 - \alpha)\sin(\omega x)e^{-\alpha t}\ ,$$

where ω and α are constants. (Make the ansatz $u(x,t) = a\sin(\omega x)e^{-bt}$.)

2. Discretize in space by using the 2nd and 4th order difference operators in Table 4.1, and also the 4th order Padé type operator in Table 4.5. Denote these operators by Q_1, Q_2, Q_3. Write a program that solves the problem by using the method

$$u_j^{n+1} = (I + kQ_\nu)u_j^n + kF_j^n\ , \qquad \nu = 1,2,3\ .$$

3. Choose $\omega = 5$ and $\alpha = 1$ and run the program with the number of points in space determined by Table 1.2. Measure the error at $t = 1$ in the max-norm, and find out if there is good agreement with the first column in the table for Q_1 and Q_2. Does Q_3 give better results than Q_2? If so, what is the explanation?

   The method is only 1st order accurate in time, which means that one has to choose a small time step, for example $k = 0.1h^2$ for Q_1 and $k = 0.01h^2$ for Q_2 and Q_3.