

Assignment 2

(Due in class Monday 25th of April before lecture)

Consider the differential equation

$$u_t + Au_x = F, \quad 0 \leq x < \infty, \quad 0 \leq t,$$

where u and F are vector functions with two components, and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

The initial and boundary conditions are

$$\begin{aligned} u(x, 0) &= f(x), \\ u^{(1)}(0, t) + \alpha u^{(2)}(0, t) &= g(t), \\ \|u\| &< \infty. \end{aligned}$$

Approximate the problem by

$$\begin{aligned} \frac{du_j}{dt} + AD_0 u_j &= F_j, \quad j = 1, 2, \dots, \\ u_j(0) &= f_j, \\ u_0^{(1)}(t) + \alpha u_0^{(2)}(t) &= g(t), \\ \frac{du_0^{(2)}}{dt} + D_+ u_0^{(1)} &= F_0^{(2)}, \\ \|u\|_h &< \infty. \end{aligned}$$

1. Derive the Lopatinsky and the Kreiss condition for the IBVP problem. Investigate when they are satisfied. For which, if any, α -values can we conclude strong well-posedness ? Note that no interval for the α -values is given.
2. Derive the Godunov-Ryabenkii and the Kreiss condition for the numerical approximation of the IBVP problem and investigate whether or not they are both satisfied for all α in the interval $0 \leq \alpha \leq 1$. For which α -values can we conclude strong stability?
3. **Challenge:** Use the normal mode analysis and derive a strongly well-posed energy-estimate for the IBVP. The estimate must contain all the data F, f, g . You can pick a suitable α if you want.

Motivate your answers clearly !