

## Assignment 1

(Due in class Monday 18th before lecture )

Consider the hyperbolic equation

$$u_t = Pu, \quad P = \frac{\partial}{\partial x}a + a\frac{\partial}{\partial x}, \quad , \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 0.5, \quad a(x) > 0, \quad (1)$$

Let  $Q$  be a semibounded difference operator satisfying

$$(v, Qv)_h \leq 0 \quad (2)$$

for all  $v$  satisfying certain boundary conditions. The so called  $\theta$ -scheme is defined by

$$(I - \theta kQ)v^{n+1} = (I + (1 - \theta)kQ)v^n. \quad (3)$$

1. Construct homogeneous (zero data) boundary conditions and a scalar product such that  $P$  is semibounded ( $(u, Pu) \leq 0$ ). Specify where the boundary condition is imposed.
2. Consider equation (1), construct discrete boundary conditions and a scalar product such that  $Qv = D_0(av) + aD_0v$  is semibounded and satisfy (2).
3. Compute the eigenvalues of  $Q$  and plot them for  $N = 20, 40, 80$ . What is the minimal real part of all the eigenvalues for each  $N$  ?
4. Run the  $\theta$ -scheme for  $\theta = 1/2, 3/4, 1$ , for  $N = 20, 40, 80$ , and demonstrate the stronger damping with increasing  $\theta$ . Use

$$\begin{aligned} a(x) &= 0.5 - 0.1x(1 - x), \\ u(x, 0) &= 1 + \sin(10\pi(x - 0.25)), \end{aligned}$$

and  $k = 2h$ .

5. **Challenge:** Prove that (3) is unconditionally stable for  $\frac{1}{2} \leq \theta \leq 1$ , and that the norm has a stronger decrease with time for larger  $\theta$  in this interval (which you should have seen in task 4. above).

Motivate your answers clearly !