Due Date: This assignment is due on Thursday, 23 April, 2009(for regular students) and on Friday, 24 April, 2009(for SCPD only, please indicate that you are SCPD), by 5pm in the box outside Durand 112. See the course website for the policy on incentives for LaTeX solutions.

Problem 1 (10 pts): Define/explain the following terms or concepts.

- Strong Law of Large Numbers
- Central Limit Theorem
- Confidence Interval
- Acceptance-Rejection RV generation

Problem 2 (10 pts): Consider a single-server queue in which customers are served according to a last in / first out non-preemptive discipline. When a given customer completes service, the server begins processing the most recently arrived customer. Customers arrive at times 1,3,5,7,11 and 15. With corresponding service requirements 4,3,1,8 and 2. How many customers are in the system at $t = 13.7$?

Problem 3 (10 pts):

1. A mean zero unit variance random variable X has a *Laplace* distribution if its p.d.f. is

$$f(x) = \frac{1}{2}e^{-|x|}.$$

   Give an algorithm to generate such random variables.

2. Using the result above, give an algorithm to generate $\mathrm{N}\left(\mu, \sigma^2\right)$ random variables.

Problem 4 (10 pts):

1. Suppose that we wish to compute $\alpha = \mathbf{E}g(W)$, where g is a non-negative function and W is a rv having density f_W. If X is a rv having positive density f_X, prove that α can be re-expressed as

$$\alpha = \mathbf{E}g(X)\frac{f_W(X)}{f_X(X)}$$

2. Prove that the variance of the Monte Carlo procedure associated with the above is minimized by choosing X to have density

$$f_X^*(x) = g(x)f_W(x)/\alpha\,.$$

3. Why is the above choice of $f_X^*(\cdot)$ impractical in general?

4. Suppose that we wish to compute $\alpha = \mathbf{P}\left\{N(0,1) > 3.75\right\}$. Compute α first by (crude) Monte Carlo sampling based on iid sampling of $\mathbb{1}_{\{N(2,1)>5.75\}}$, and then compute α via the approach suggested in 3. with density

$$f_X(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-(x-5.75)^2/2\right)$$

   (i.e. sampling X according to a $N(5.75, 1)$ distribution). Produce a 90% confidence interval based on 10000 and 100000 samples for each approach.

   Remark. The Monte Carlo approach described above is called "importance sampling".