

## CME 305: Discrete Mathematics and Algorithms

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Midterm – 02/17/09

**Problem 1.** We say that a graph is  $k$ -regular if all its vertices have degree  $k$ . Prove that all  $k$ -regular bipartite graphs have a perfect matching.

Hint: use Hall's Marriage Theorem

**Problem 2.** A proper coloring of the edges of a graph  $G$  is an assignment of colors to edges such that no two adjacent edges have the same color. A graph is said to be  $k$ -edge-colorable if there is a proper edge-coloring of the graph that uses  $k$  or fewer colors.

Prove that if the maximum degree in a bipartite graph  $G$  is  $\Delta$ , then  $G$  is  $\Delta$ -edge-colorable. Also provide a polynomial-time algorithm that produces such a coloring.

**Problem 3. (extra credit)** Give a polynomial time algorithm for a proper coloring of the vertices of a 3-vertex-colorable graph of size  $n$  with at most  $O(\sqrt{n})$  colors.