CME 305: Discrete Mathematics and Algorithms

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HW#3 – Due 03/12/09

1. Kleinberg and Tardos Problem 8.28

2. Let A be a $n \times n$ matrix, B a $n \times n$ matrix and C a $n \times n$ matrix. We want to design an algorithm that checks whether $AB = C$ without calculating the product AB. Provide a randomized algorithm that accomplishes this in $O(n^2)$ time with high probability.

3. A tournament is a complete directed graph i.e. a directed graph which has exactly one edge between each pair of vertices. A Hamiltonian path is a path that traverses each vertex exactly once. A random tournament, is a tournament in which the direction of all edges is selected independently and uniformly at random.

   (a) What is the expected value of the number Hamiltonian paths in a random tournament?

   (b) Use part (a) to show that for every n, there is a tournament with n players and at least
   $$\frac{n!}{2^{n-1}}$$
   Hamiltonian paths.

4. In class we proved that the presorted Longest Processing Time (LPT) minimum makespan approximation algorithm gives a 3/2-approximation, but noted that this is not the best bound possible.

   Show that LPT is a 4/3-approximation. Give an example to show this bound is tight.

5. Kleinberg and Tardos Problem 11.10

6. The final expectations for the project are (1) a literature survey, (2) understanding of chosen topic and an exposition of current state of the art (3) some type of extension e.g. coding up an algorithm, proving a lemma missing from the paper, solving an open problem etc...

   Submit a small write up (one per group) describing what your extension will be.

7. Extra Credit: Solve Problem 3 from the midterm.