

## 2011 CME304/MS&E315 Class Project

Queen Dido, left her home in Tyre to escape from her brother. Lengthy is her tale of wrong, lengthy the windings of its course; but I will pass rapidly from point to point. Her husband was Sychaeus, wealthiest of Phoenician landowners, and loved by his poor wife with fervid passion; on him her father had bestowed her in her maiden bloom, linking them together by the omens of a first bridal. But the crown of Tyre was on the head of her brother, Pygmalion, in crime monstrous beyond the rest of men. They were two, and fury came between them. Impious that he was, at the very altar of the palace, the love of gold blinding his eyes, he surprises Sychaeus with his stealthy steel, and lays him low, without a thought for his sister's passion; he kept the deed long concealed, and with many a base coinage sustained the mockery of false hope in her pining love-lorn heart. But lo! in her sleep there came to her no less than the semblance of her unburied spouse, lifting up a face of strange unearthly pallor; the ruthless altar and his breast gored with the steel, he laid bare the one and the other, and unveiled from first to last the dark domestic crime. Then he urges her to speed her flight, and quit her home for ever, and in aid of her journey unseals a hoard of treasure long hid in the earth, a mass of silver and gold which none else knew. Dido's soul was stirred; she began to make ready her flight, and friends to share it. There they meet, all whose hate of the tyrant was fell or whose fear was bitter; ships, that chanced to lie ready in the harbour, they seize, and freight with gold. Away it floats over the deep, the greedy Pygmalion's wealth; and who heads the enterprise? a woman! So they came to the spot where you now see yonder those lofty walls, and the rising citadel of Carthage the new; there they bought ground, which got from the transaction the name of Byrsa, as much as they could compass round with a bull's hide.

— Virgil, *The Aeneid*, translated by John Conington

**Part I.** Queen Dido has enlisted you, a student of Numerical Optimization, to maximize the net value of land obtained through this transaction. Dido is an industrious figure and has already cut the bull's hide into  $n$  straight strips of  $l$  podes. These strips connected end-to-end and intersecting the linear coastline at two points will define the boundary of Byrsa. A square pous of land is valued at  $p_a$  drachmas. Dido, well versed in ancient Mediterranean real estate, adds an additional value to land along the coastline at  $p_c$  drachmas per pous.

**Part II.** Having solved the problem, Dido realizes had she used  $2n$  strips of  $\frac{l}{2}$  podes the value of the land could have been increased. How do you advise Dido solve the new problem making use of the old solution?

**Multivariate optimization routine.** You will write a routine to solve the penalized bound-constrained optimization problem

$$\begin{aligned} & \text{minimize } F(x) \\ & \text{subject to } l \leq x \leq u. \end{aligned}$$

You are free to choose the programming language and algorithm. Your model will likely involve a set of equality constraints,  $c(x) = 0$ . These may be eliminated with the penalty method by defining the objective function as  $F_\rho(x) = F(x) + \frac{\rho}{2}c(x)^T c(x)$ . You will then solve a sequence of optimization problems with increasing  $\rho$  to satisfy the constraints.

**Report.** Your report should describe the following:

- each algorithm;
- your model;
- how you verified your software is working properly;
- how you verified your solution is correct;
- clear, neat, and informative output;
- extensions you explored.

You should not include your source code unless an excerpt improves the clarity of your report. L<sup>A</sup>T<sub>E</sub>X is required. Please include a bibliography.

**Comments.**

- Different formulations will have algorithmic and numerical consequences. It is worthwhile to work out different models and consider their various properties.
- Experiment with different values of the parameters  $n$ ,  $l$ ,  $p_c$ , and  $p_a$ . It is often the case that algorithm performance degrades under finer discretizations. What happens as you increase  $n$ ? How large a problem are you able to solve?
- It is generally a good idea to develop and test your routines in small pieces. Get linesearch working before your optimization code. Use simple test functions for verification.
- This is an individual project. You may not collaborate with anyone.