The Problem

 $\begin{array}{ll} \underset{x}{\operatorname{minimize}} & f(x) & \text{objective} \\ \text{subject to} & l \leq x \leq u & \text{simple bounds} \\ & l_A \leq Ax \leq u_A & \text{linear constraints} \\ & l_C \leq c(x) \leq u_C & \text{nonlinear constraints} \end{array}$

where x, l, u are in \mathbb{R}^n , A is an $m_1 \times n$ matrix, c(x) is an m_2 -vector of smooth nonlinear functions and f(x) is a smooth scalar function.

Placement of Substations in an Electrical Network (SSO)

Electric Utility Substation Risk Analysis and Planning Orange County Emergency Management









Service area: each grid block is 1/2 mile by 1/2 mile

SSO - Review

- "Model distribution lines and substation locations and
 - Determine the optimal substation capacity additions
 - To serve a known load at a minimum cost"



Service area: each grid block is 1/2 mile by 1/2 mile

SSO - Review

Characteristics:

More substations: Higher capital cost Lower transmission cost

Capital costs: \$4,000,000 for a 28 MW substation

Cost of losses: \$3,000 per kw of losses



Service area: each grid block is 1/2 mile by 1/2 mile

Variables

- \overline{I} : Vector of currents at each node in the network
- V: Vector of voltages at each node in the network
- Y: Network Admittance matrix
- y_i : Binary variable specifying whether node *i* has a substation or not.
- C_{cap} : The installation cost of a substation.
- C_{loss} : The cost of electrical losses in a network.
- S_{cap} : Capacity of a substation.
- ℓ_i : Electrical load at node i.
- $m \times n$: Grid dimension.

Problem of Interest

SSO	$\underset{V,\overline{I},y}{\operatorname{minimize}}$	$C_{cap}e^T y + C_{loss}V^T Y V$
		$\overline{I} - YV = 0$
		$V^l \leq V_i \leq 1$ for all i
	subject to	$\overline{I_i} \leq S_{cap}, y_i = 1 \text{ for all } i$
		$\overline{I_i} = \ell_i, y_i = 0$ for all i
		$y_i \in \{0,1\}$ for all i

Admittance Matrix



Y is symmetric and positive semidefinite.

Y has one zero eigenvalue with an associated eigenvector (1, ..., 1)

 A_i and B_i are tridiagonal and diagonal matrices, respectively.

OPTIMIZATION OF LIBRATION POINT MISSION TRAJECTORIES

- At a libration or Lagrange point the forces of two orbiting celestial bodies balance to zero. The two points for the Sun-Earth system that are colinear are known as the L₁ and L₂ points (there are 5 points altogether). In theory if a body was placed at such a point it would remain. The trick is to put it there. At such points no energy is needed to loiter forever, which is what makes it attractive for long term missions. The points are also sufficiently close to Earth that communications is not a issue.
- In reality the forces are more complex and are not independent of time, which implies not only will the point move but the movement leaves any satellite in the old position subject to force and it will also move and continue to move.

Background



Background



James Webb Space Telescope





Equations of Motion

$$\begin{aligned} \ddot{x} &= 2\dot{y} + x - \frac{(1-\mu)(x-\mu)}{r_{13}^3} - \frac{\mu(x+1-\mu)}{r_{23}^3} + F_x/m \\ \ddot{y} &= -2\dot{x} + y - \frac{(1-\mu)y}{r_{13}^3} - \frac{\mu y}{r_{23}^3} + F_y/m \\ \ddot{z} &= -\frac{(1-\mu)z}{r_{13}^3} - \frac{\mu z}{r_{23}^3} + F_z/m, \end{aligned}$$

Small Halo



Loitering



1.1 The Frequency Assignment Problem (FAP)

PROBLEM FORMULATION:

Find an assignment of frequencies to base stations that minimizes an objective and fulfills a set of side constraints.

-NP hard

F1 F2

F3 F4



1.5 Characteristics of FAP Data

int. rel.



Geographic distribution of base station locations

Distribution of interf. relations in a data set (e.g. in a given region)

Interference [0.1%]

Equivalence with Global Optimization

• Transformation of integer requirements to constraints

$$0 \le x_j \le u_j, x_j \in \mathbf{Z} \iff \prod_{i=1}^{\lfloor u_j \rfloor} (x_j - i) = 0$$

Addition of Penalty Function

Minimize
$$f(x) + \gamma \sum_{j=1}^{n} \prod_{i=1}^{\lfloor u_j \rfloor} (x_j - i)$$

subject to $Ax = b$

Need to find global optimizer of above problem



Phase Configuration Ariane 5 Ascent





Reentry Application:

Hermes Reentry, maximum crossrange





Direct optimization of Step-and-Shoot segments

Anning Magnet

Wedge Block Filters Collimator

ble Focused ti-Leaf Collimator



Numerical Optimization



Radiating tumors

SNOPT having an impact



Conventional Launcher: Ariane 5 Dual Payload LEO/GEO



Robot at JPL



Facility location: substations in a network

Numerical Optimization



Radiating tumors

SNOPT having an impact



Repositioning the space station



Robot at JPL



Facility location: substations in a network

