Math 131 Final examination

December 14, 2005

Name______________________________________

Student ID__________________________________

Signature___________________________________

1	
2	
3	
4	
Total	

Directions:

1. This is closed books / notes exam (see attached formula sheet at the last page).
2. Your signature above indicates that you accept the University Honor Code.
3. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown will receive no credit.
4. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
5. This test is 2 hours long and has 4 problems.
6. Good luck!

Problem 1 Given PDE

$$v_{tt} - v_{xx} - 4v_t - 2v_x + 3v = 0\,, \quad x, t \in (0, \infty)\,.$$

(a) (20 points) Find its general solution

Hint: Find the change of variables of the type $v(x,t) = e^{ax+bt}u(x,t)$ with the constants a, b that reduces PDE to the wave equation.

(b) (5 points) Find the explicit formula for a solution of this PDE, which satisfies the following initial conditions:

$$v(x,0) = 0\,, \quad v_t(x,0) = x\,.$$

Problem 2 (25 points) Find the integral formula for a solution $v(x,t)$ of the system

$$\begin{array}{ll} DE & v_t = k v_{xx}\,, \quad x \in (0,\infty);\; t \in (0,\infty) \\ IC & v(x,0) = 1 \\ BC & v(0,t) = t + 1\,. \end{array}$$

Hint: Make a substraction $u(x,t) = v(x,t) - g(t)$. What would be a good choice for $g(t)$?

Problem 3 (30 points) Find the solution in series form (explicitly compute the coefficients) of

$$\begin{array}{cl} PDE & u_{tt} - u_{xx} - u_t = 0\,, \quad x \in (0,\pi);\ t \in (0,\infty) \\ IC & u(x,0) = 1,\ u_t(x,0) = 0 \\ BC & u(0,t) = u(\pi,t) = 0\,. \end{array}$$

Hint: Use the method of separation of variables.

Problem 4 The (unscaled) Legendre polynomials $P_n(x)$ are defined as

$$P_0(x) = 1\,, \quad P_n(x) = \frac{d^n}{dx^n}\left[(x^2-1)^n\right]\,,\ n = 1, 2, 3\ldots$$

(a) (15 points) Prove that the Legendre polynomials are orthogonal on the interval $-1 \leq x \leq 1$, that is $\int_{-1}^{1} P_n(x)P_m(x)dx = 0$ unless $n = m$.
Hint: Use integration by parts. How many times one has to perform it, and in which direction?

(b) (5 points) Compute A_0, A_1 in the general Fourier series $\sum_{n=0}^{\infty} A_n P_n(x)$ for a function $f(x) = \sin(x)$ on the interval $[-1, 1]$.