Lecture 8 - SISO Loop Design

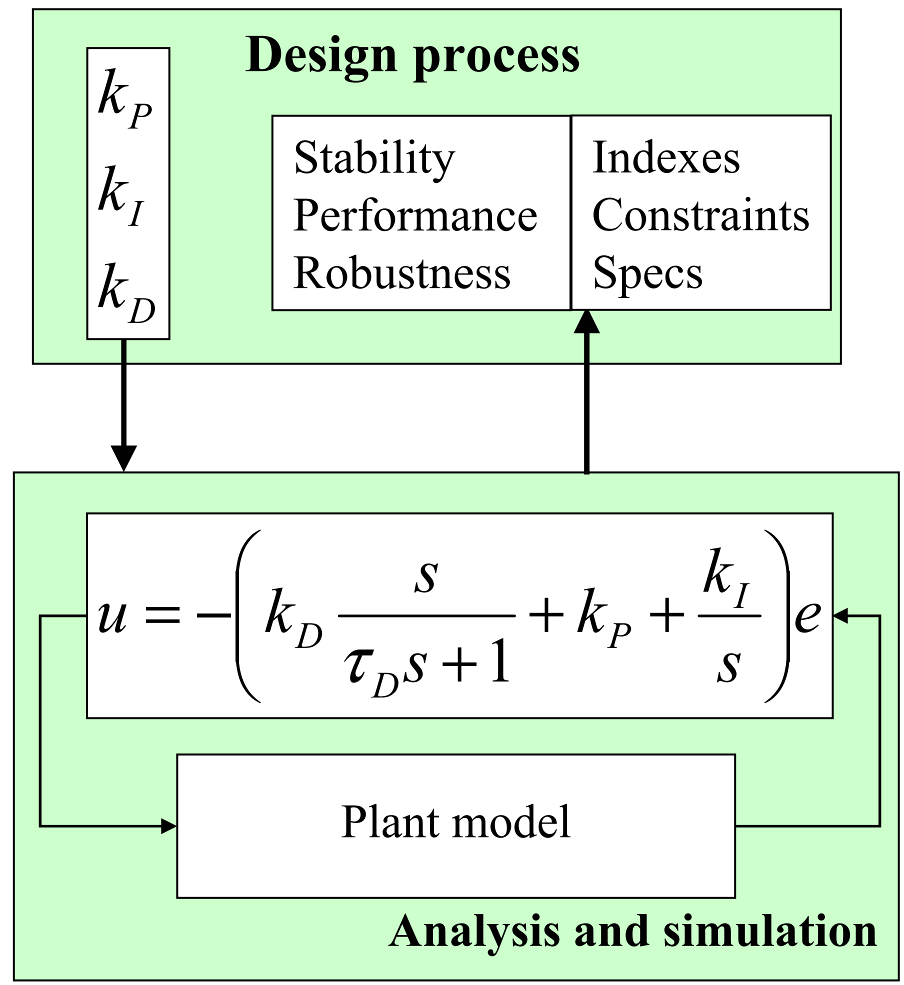
- Design approaches, given specs
- Loopshaping: in-band and out-of-band specs
- Fundamental design limitations for the loop

Modern Control Theory

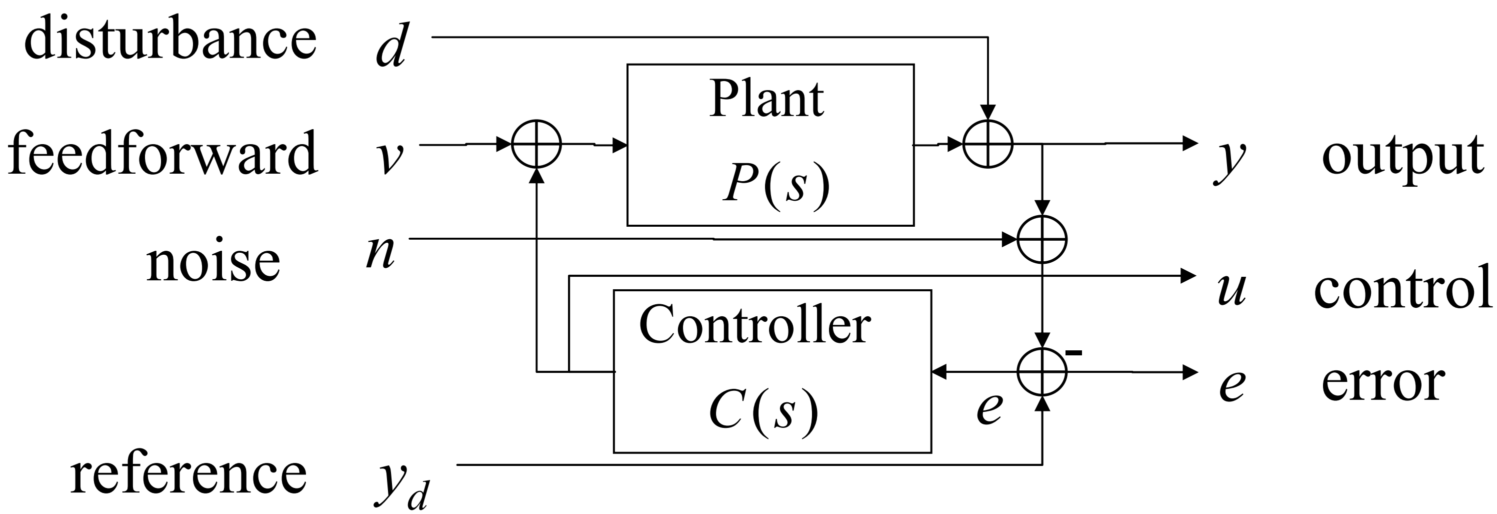
- Appy results of EE205 etc
- Observable and controllable system
  - Can put poles anywhere
  - Can drive state anywhere
- Why cannot we just do this?
  - Large control
  - Error peaking
  - Poor robustness, margins
    - Observability and controllability = matrix rank
    - Accuracy of solution is defined by condition number
- Analysis is valid for *any* LTI control, including advanced

Feedback controller design

- Conflicting requirements
- Engineers look for a reasonable trade-off
  - Educated guess, trial and error controller parameter choice
  - Satisfy key specs
  - Find a reasonable tradeoff between several conflicting requirements



Transfer functions in control loop



- Sensitivity
- Complementary sensitivity
- Noise sensitivity
- Load sensitivity

$$S(s) = [1 + P(s)C(s)]^{-1}$$

$$T(s) = [1 + P(s)C(s)]^{-1} P(s)C(s)$$

$$S_u(s) = [1 + P(s)C(s)]^{-1} C(s)$$

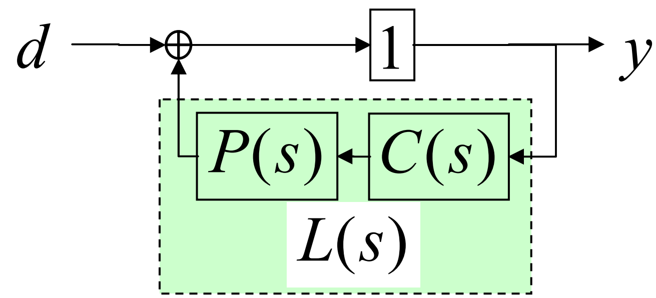
$$S_y(s) = [1 + P(s)C(s)]^{-1} P(s)$$

$$\boxed{S(s) + T(s) = 1}$$

Loop shape performance requirements

Loop gain: $L(i\omega) = P(i\omega)C(i\omega)$

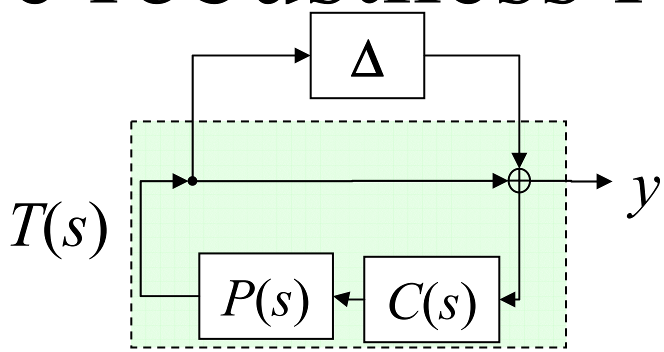
$$S(i\omega) = \left[1 + L(i\omega)\right]^{-1}$$

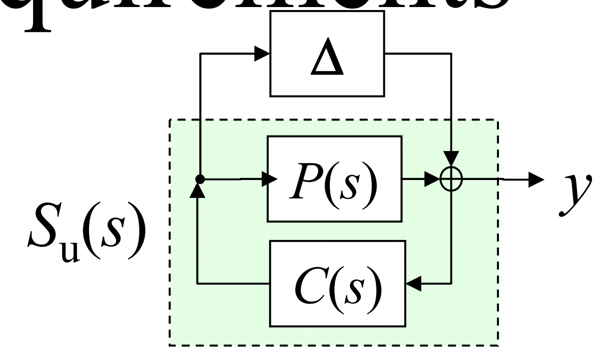


Performance

- Disturbance rejection and reference tracking
  - $|S(i\omega)| << 1$ for the disturbance d;
  - **satisfied if $|L(i\omega)| >> 1$**
- Noise rejection
  - $|T(i\omega)| = |[1 + L(i\omega)]^{-1} L(i\omega)| < 1$
  - is satisfied unless $|1 + L(i\omega)|$ is small (near the crossover)
- Limited control effort
  - $|C(i\omega)\, S(i\omega)| < 1$
  - Can be a problem if $|P(i\omega)| < 1$ (high frequency)

Loop shape robustness requirements





Robustness

- Multiplicative uncertainty
  - $|T(i\omega)|\cdot|\Delta(i\omega)| < 1$, where $|\Delta(i\omega)|$ is the uncertainty magnitude
  - at high frequencies, relative uncertainty $|\Delta(i\omega)|$ can be large, hence, $|T(i\omega)|$ must be kept small
  - **must have $|L(i\omega)| \ll 1$ for high frequency, where $|\Delta(i\omega)|$ is large**
- Additive uncertainty
  - $|C(i\omega)\, S(i\omega)| < 1/\,|\Delta(i\omega)|$
- Gain margin of 10-12db and phase margin of 45-50 deg
  - this corresponds to relative uncertainty of the plant transfer function in the 60-80% range around the crossover

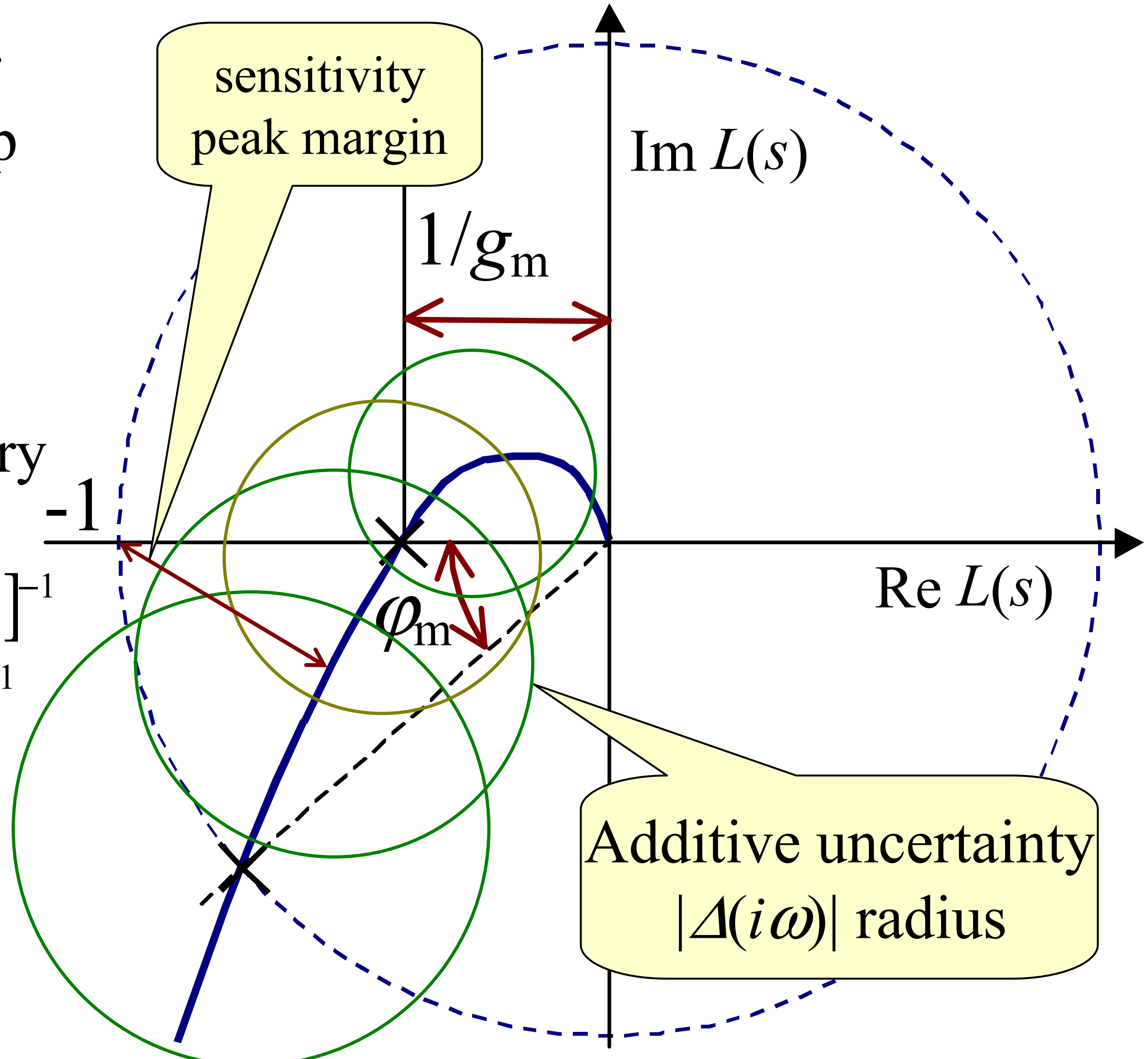
Sensitivity vs. margins

- Margins are useful for deciding upon the loop shape modifications
- Can use uncertainty characterization and noise or complementary sensitivity instead

$$S_u(s) = C(s)[1 + P(s)C(s)]^{-1}$$

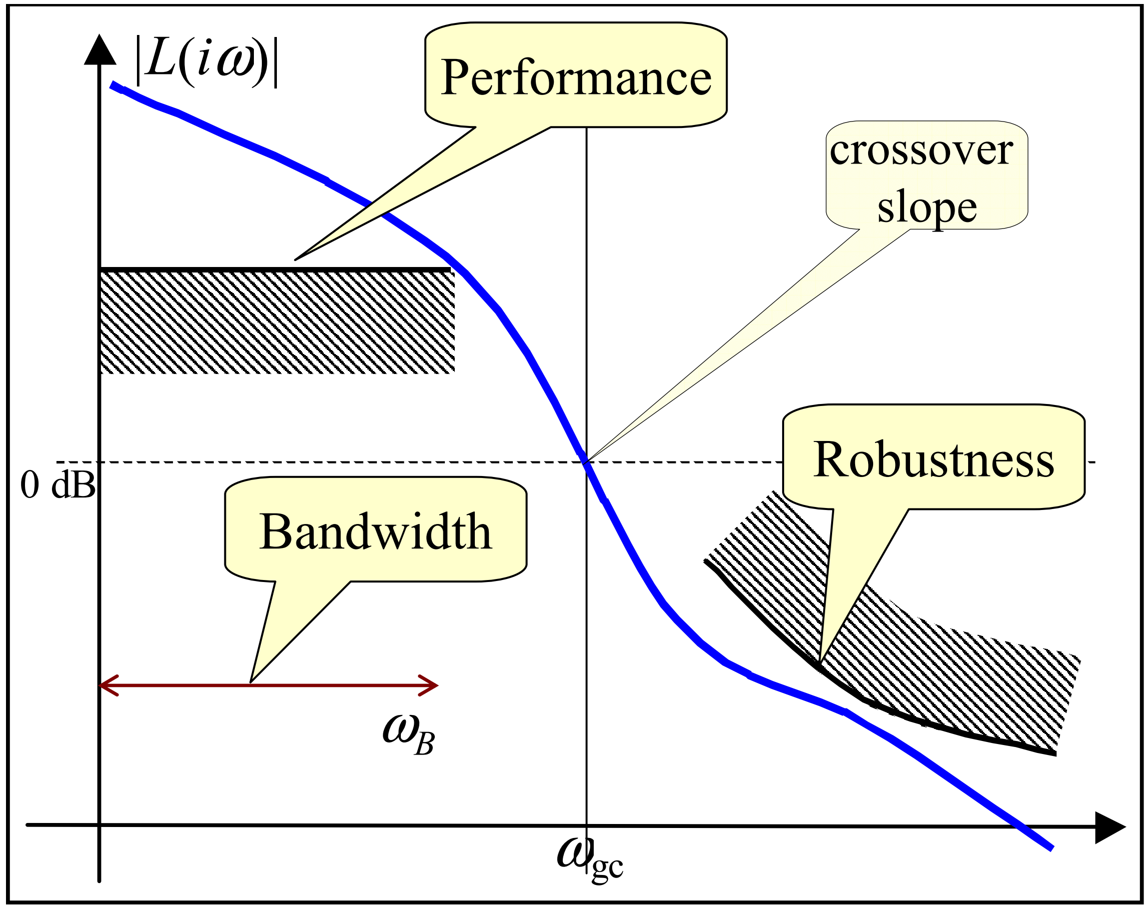
$$T(s) = 1 - [1 + P(s)C(s)]^{-1}$$

- This is done by modern advanced control design methods



Loop Shape Requirements

- Low frequency:
  - high gain L = small S
- High frequency:
  - small gain L → small T · large Δ
- Bandwidth
  - performance can be only achieved in a limited frequency band: $\omega \leq \omega_B$
  - ω_B is the bandwidth



Fundamental tradeoff: performance vs. robustness

Loopshaping design

- Loop design
  - Use P,I, and D feedback to shape the loop gain
- Loop modification and bandwidth
  - Low-pass filter - get rid of high-frequency stuff - robustness
  - Notch filter - get rid of oscillatory stuff - robustness
  - Lead-lag to improve phase around the crossover - bandwidth
    - P+D in the PID together have a lead-lag effect
- Need to maintain stability while shaping the magnitude of the loop gain
- Formal design tools H_2, H_∞, LMI, H_∞ loopshaping
  - cannot go past the fundamental limitations

Example - disk drive servo

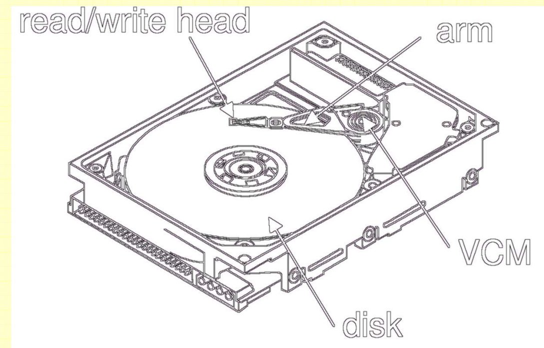
- The problem from HW Assignment 2
  - data in `diskPID.m, diskdata.mat`
- Design model: $\Delta P(s)$ is an uncertainty

$$P(s) = \frac{g_0}{s^2} + \Delta P(s)$$

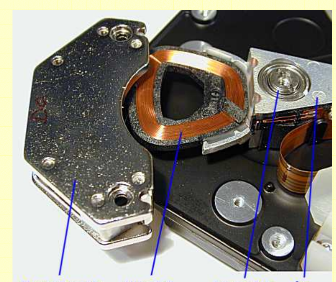
- Analysis model: description for $\Delta P(s)$
- Design approach: PID control based on the simplified model

$$C(s) = k_P + \frac{k_I}{s} + k_D \frac{s}{\tau_D s + 1}$$

Disk servo control



$$J\ddot{\varphi} = T_{VCM} + T_{DISTURBANCE}$$



Voice Coil Motor

Disk drive servo controller

- Start from designing a PD controller
  - poles, characteristic equation

$$1 + C(s)P(s) = 0 \Rightarrow \left(k_P + sk_D\right) \cdot \frac{g_0}{s^2} + 1 = 0$$

$$s^2 + sg_0k_D + g_0k_P = 0$$

- Critically damped system

$$k_D = 2w_0 / g_0; \qquad k_P = w_0^{\ 2} / g_0$$

where frequency w_0 is the closed-loop bandwidth

- In the derivative term make dynamics faster than w_0. Select $\tau_D = 0.25 / w_0$

$$k_D \frac{s}{\tau_D s + 1}$$

Disk drive servo

- Step up from PD to PID control

$$1 + \left(k_P + s k_D + \frac{1}{s} k_I \right) \cdot \frac{g_0}{s^2} = 0$$

$$s^3 + s^2 g_0 k_D + s g_0 k_P + g_0 k_I = 0$$

- Keep the system close to the critically damped, add integrator term to correct the steady state error, keep the scaling

$$k_P = w_0^2 / g_0; \quad k_D = a w_0 / g_0; \quad k_I = b w_0^3 / g_0 \quad \tau_D = c / w_0$$

where *a, b,* and *c* are the tuning parameters

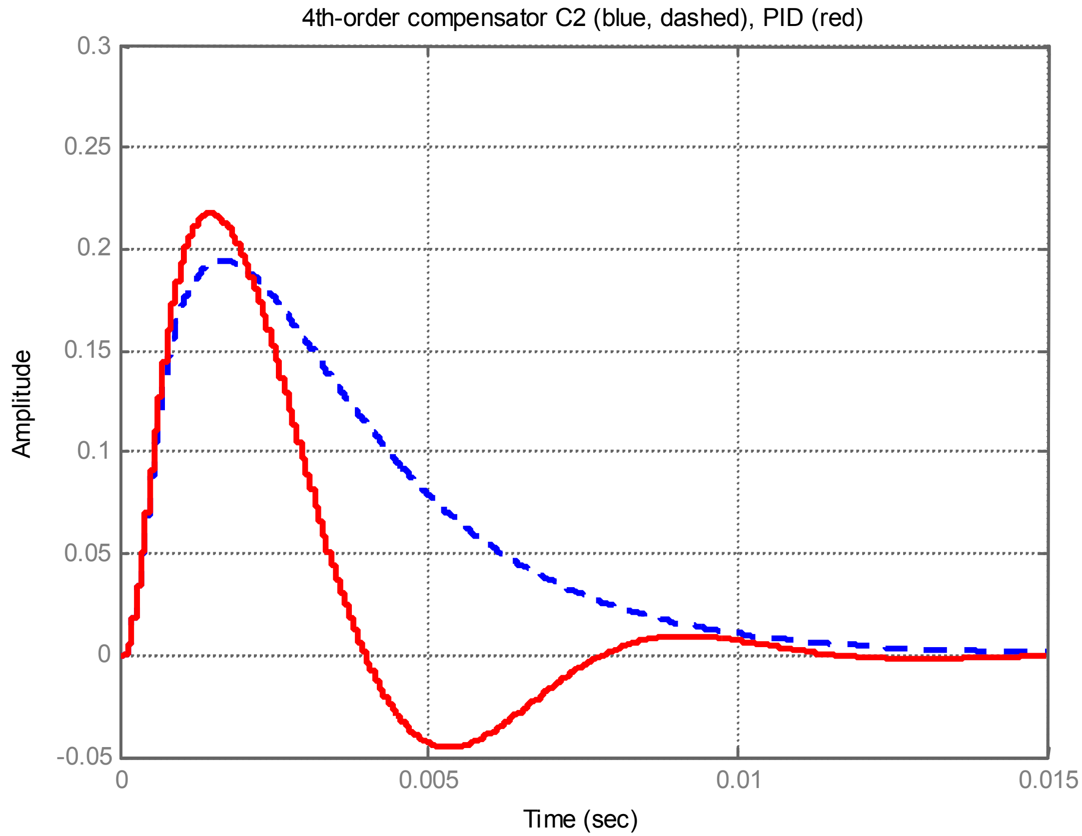
- Tune *a, b, c* and w_0 by watching performance and robustness

Disk drive - controller tuning

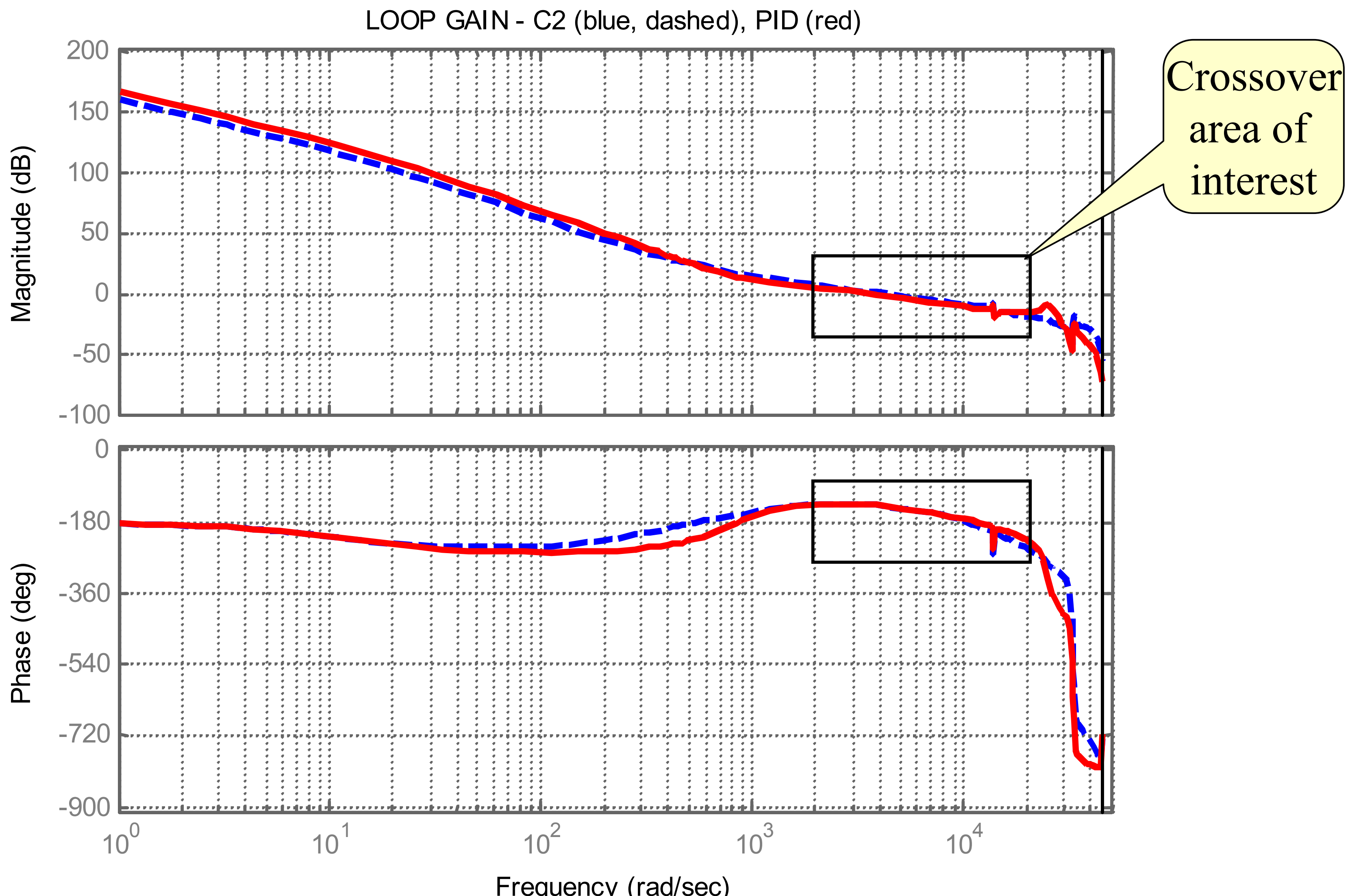
- Tune *a, b,* w_0 , and τ_D by trial and error
- Find a trade off taking into the account
  - Closed loop step response
  - Loop gain - performance
  - Robustness - sensitivity
  - Gain and phase margins
- Try to match the characteristics of C2 controller (demo)

Disk servo - controller comparison

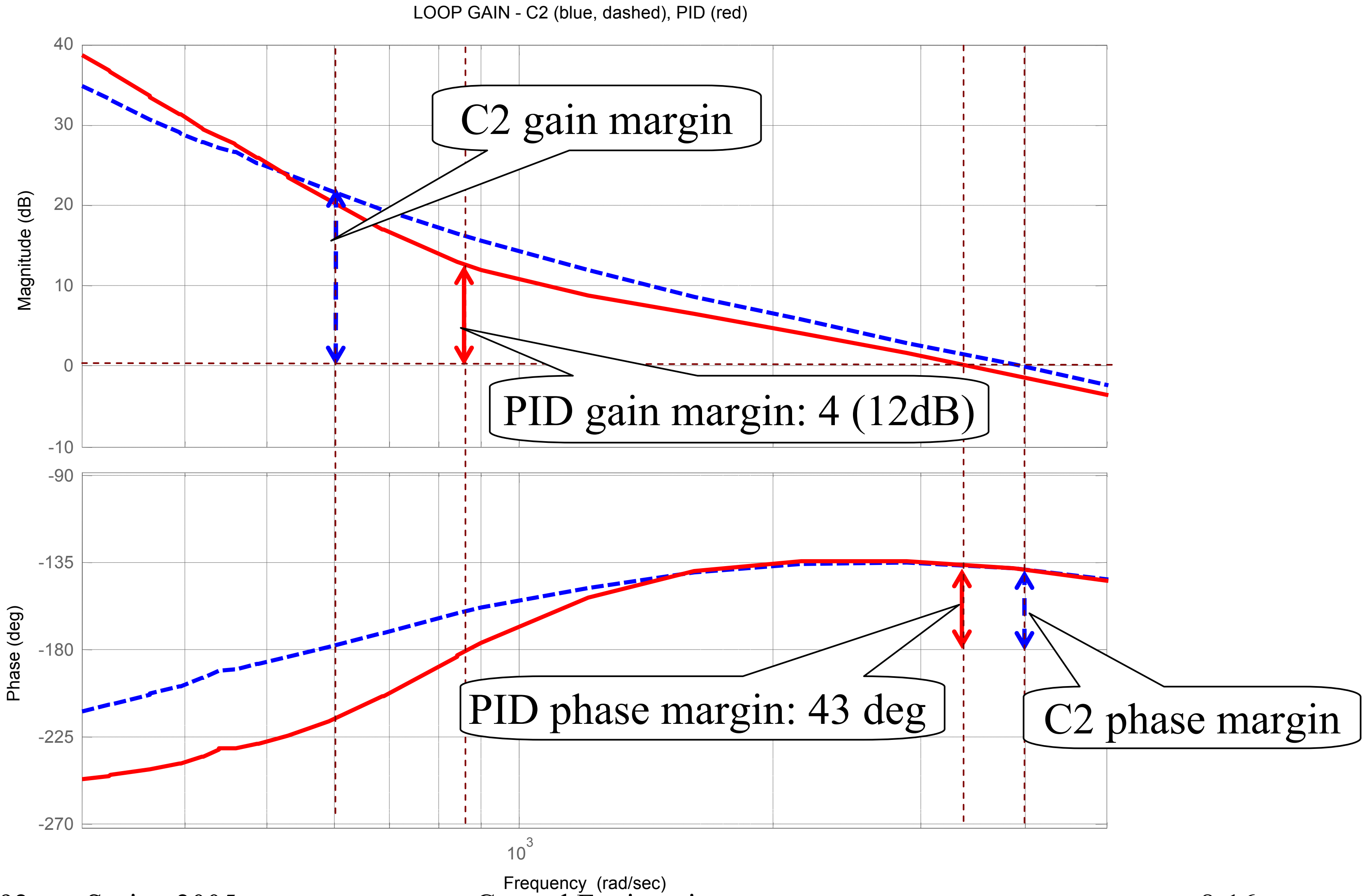
- PID is compared against a reference design
- Reference design: 4-th order controller C2 = lead-lag + notch filter
  - Matlab `diskdemo`
  - Data in `diskPID.m`, `diskdata.mat`



Loop shape, margins



Loop shape and margins, zoomed



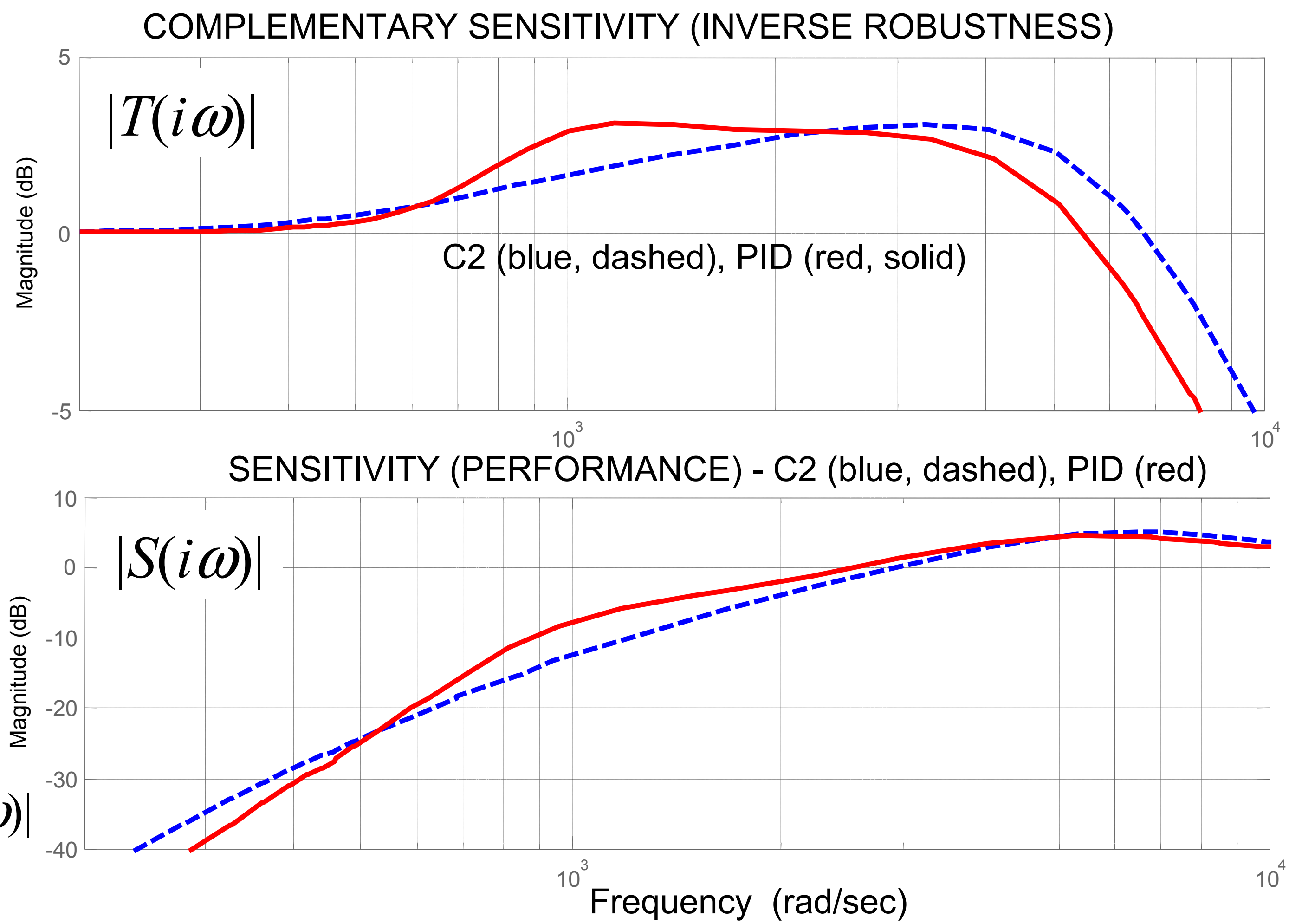
Sensitivities

Robustness to multiplicative uncertainty

$$|T(i\omega)|\cdot|\Delta(i\omega)| < 1$$

Disturbance rejection performance

$$|y(i\omega)| = |S(i\omega)|\cdot|d(i\omega)|$$



Fundamental design limitations

- If we do not have a reference design - how do we know if we are doing well. Is there is a much better controller?
- Cannot get around the fundamental design limitations
  - frequency domain limitations on the loop shape
  - system structure limitations
  - engineering design limitations

Frequency domain limitations

- Performance vs. robustness tradeoff

Disturbance rejection performance: $|S(i\omega)| << 1$

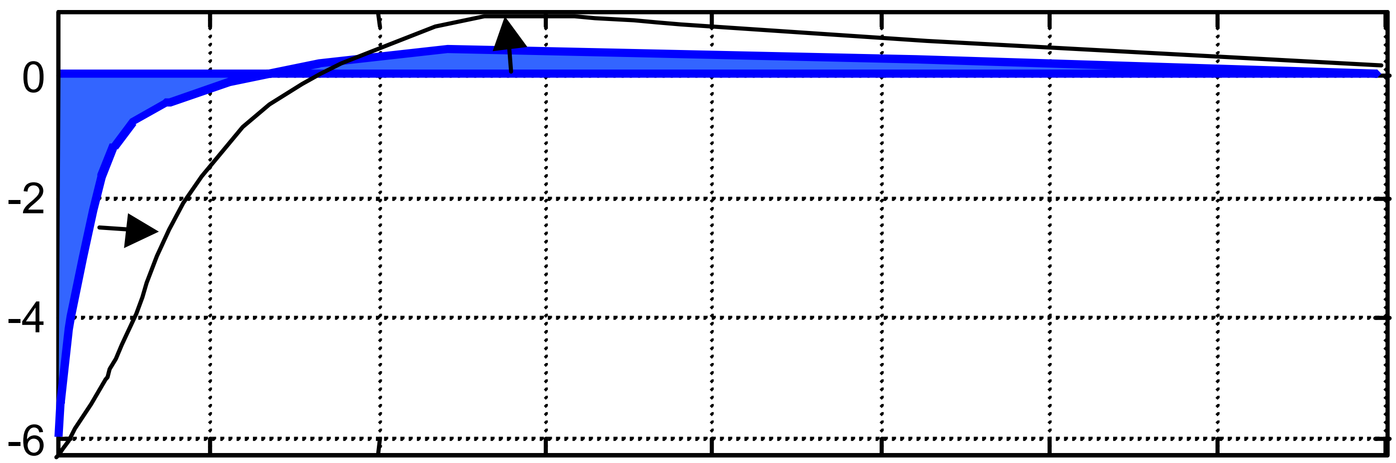
$$S(i\omega) + T(i\omega) = 1$$

Robustness: $|T(i\omega)| << 1$

- Bode's integral constraint - waterbed effect

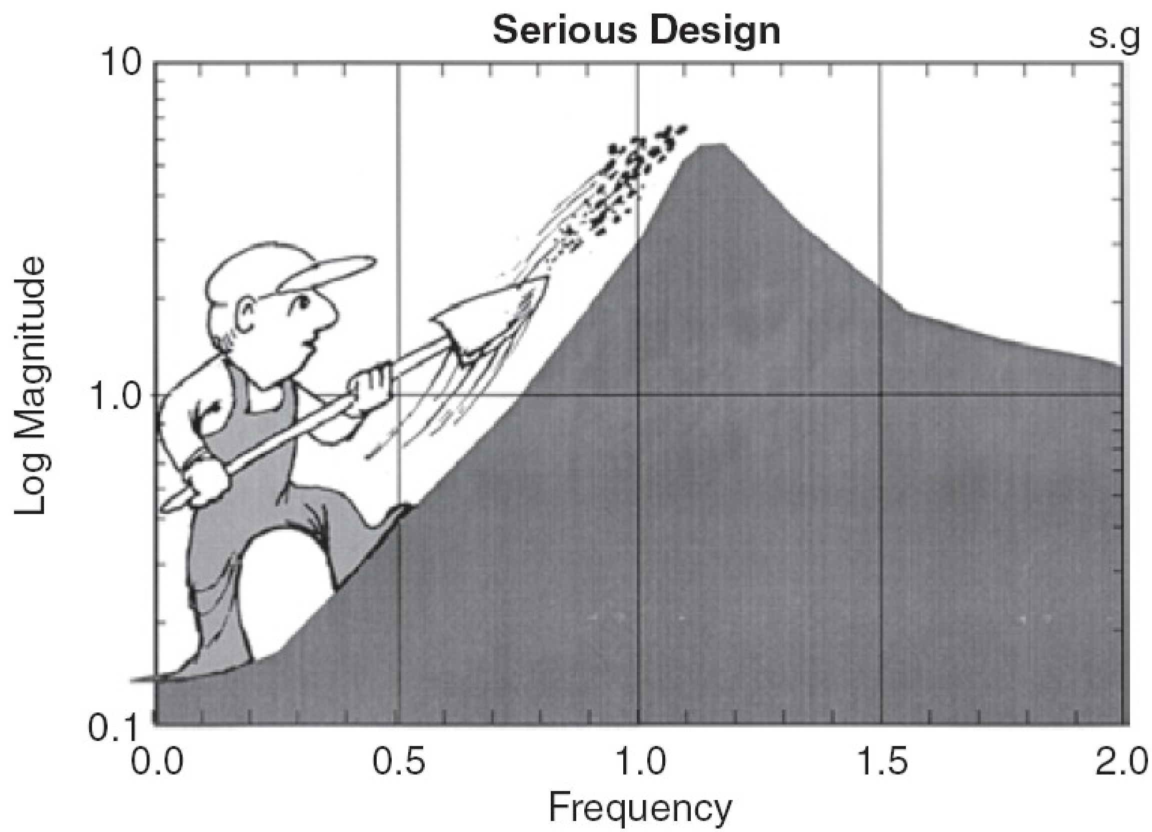
$$\int_0^\infty \log|S(i\omega)|\,d\omega = 0$$ for minimum-phase stable systems; worse for the rest

log $|S(i\omega)|$

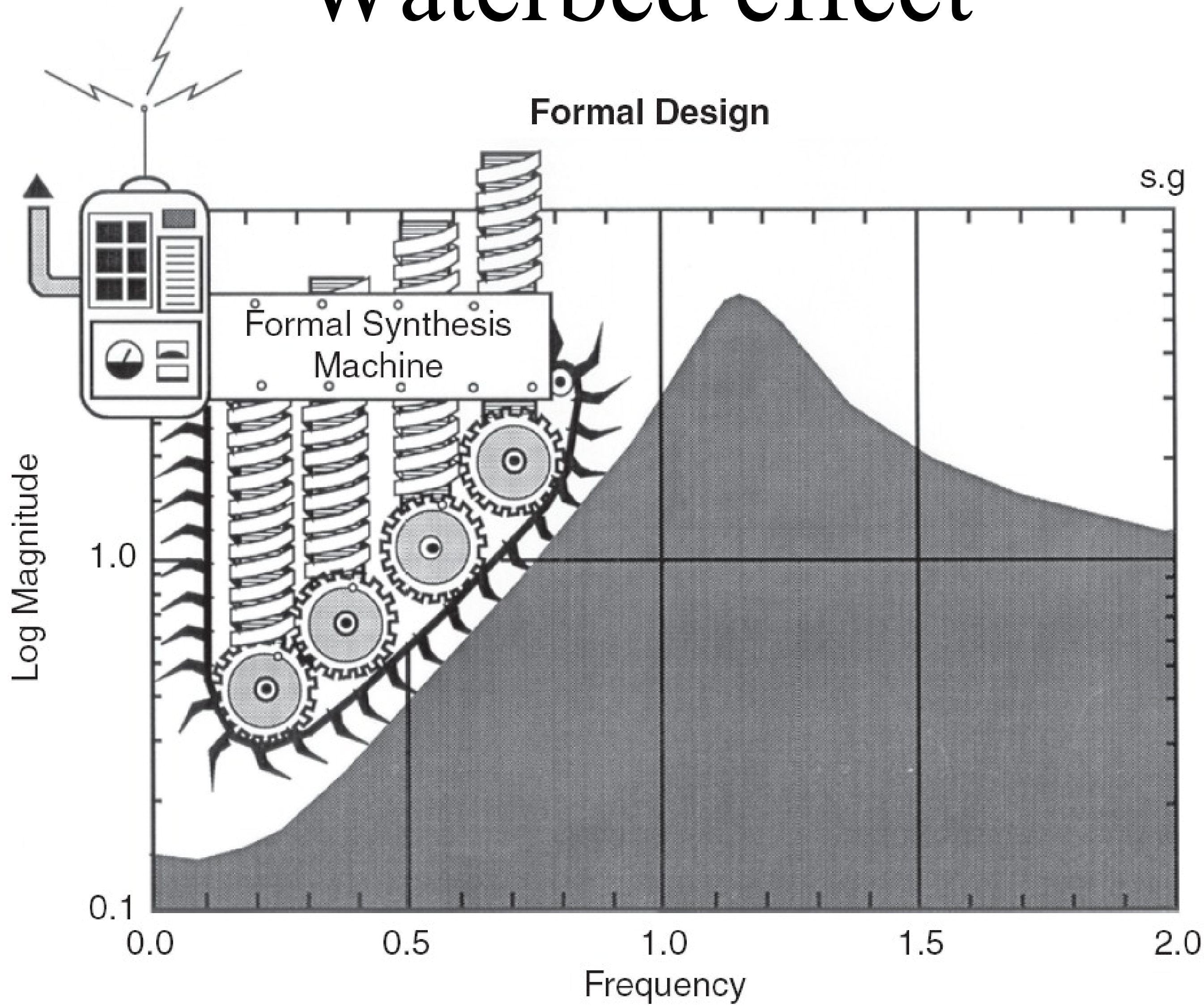


Waterbed effect

- Gunter Stein's Bode Lecture, 1989 (IEEE CSM, August 2003)



Waterbed effect

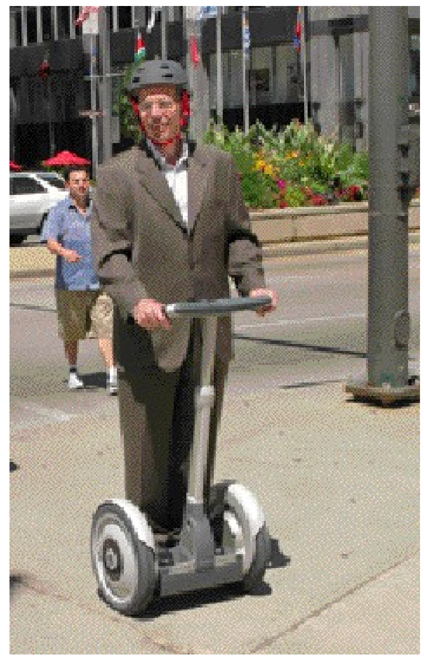


Structural design limitations

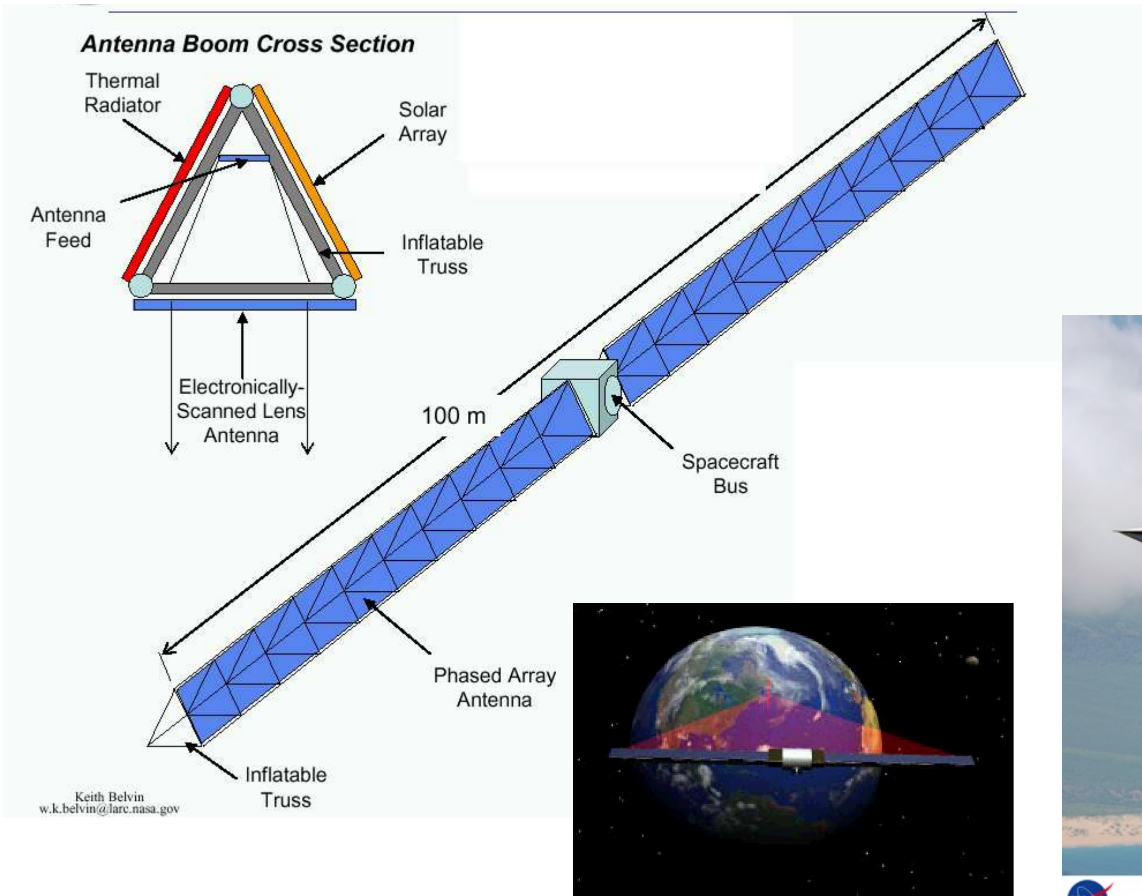
- Delays and non-minimum phase (r.h.s. zeros)
  - cannot make the response faster than delay, set bandwidth smaller
- Unstable dynamics
  - makes Bode's integral constraint worse
  - re-design system to make it stable or use advanced control design
- Flexible dynamics
  - cannot go faster than the oscillation frequency
  - practical approach:
    - filter out and use low-bandwidth control (wait till it settles)
    - use input shaping feedforward

Unstable dynamics

- Advanced applications
  - need advanced feedback control design



Flexible dynamics



- Very advanced applications
  - really need control of 1-3 flexible modes

Engineering design limitations

- Sensors
  - noise - have to reduce $|T(i\omega)|$ - reduced performance
  - quantization - same effect as noise
  - bandwidth (estimators) - cannot make the loop faster
- Actuators
  - range/saturation - limit the load sensitivity $|C(i\omega)\ S(i\omega)|$
  - actuator bandwidth - cannot make the loop faster
  - actuation increment - sticktion, quantization - effect of a load variation
  - other control handles
- Modeling errors
  - have to increase robustness, decrease performance
- Computing, sampling time
  - Nyquist sampling frequency limits the bandwidth