Lecture 3 – Basic Feedback

Simple control design and analysis

- Linear model with feedback control
- Use simple model → design control → validate
- Simple P loop with an integrator
- Velocity estimation
- Time scale
- Cascaded control loops

Feedback Stability – State Space

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx$$

Simple feedback control

$$u = -K(y - y_d)$$

- Closed-loop dynamics

$$\frac{dx}{dt} = A_K x + B_K y_d$$

$$A_K = A - BKC$$
$$B_K = BK$$

- Stability is described by the closed-loop poles

$$\{\lambda_j\} = \mathrm{eig}(A_K)$$

Closed-loop eigenvalues

F16 Longitudinal Model Example

$$A = \begin{bmatrix} -1.93\cdot 10^{-2} & 8.82 & -32.2 & -0.48 \\ -2.54\cdot 10^{-4} & -1.02 & 0 & 0.91 \\ 0 & 0 & 0 & 1 \\ 2.95\cdot 10^{-12} & 0.82 & 0 & 1.08 \end{bmatrix} \qquad B = \begin{bmatrix} 0.17 \\ -2.15\cdot 10^{-3} \\ 0 \\ -0.18 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 57.3 & 0 \end{bmatrix}$$

- Roots = poles = eigenvalues
  $\text{eig}(A - BKC)$
- Can be plotted for different gains K
- Root locus plot

```
% Take A from
% the F16 example
>> eig(A)

ans =
   -1.9125
   -0.1519 + 0.1143i
   -0.1519 - 0.1143i
    0.0970
```

```
% Closed-loop poles
>> K = -0.2;
>> eig(A-B*K*C)

ans =
   -1.4419
   -0.0185
   -0.3294 + 1.1694i
   -0.3294 - 1.1694i
```

Closed-loop poles

- Transfer function poles tells you everything about stability
- Model-based analysis for a simple feedback example:

$$y = H(s)u \quad u = -K(y - y_d) \quad \Longrightarrow \quad y = \frac{H(s)K}{1 + H(s)K} y_d = L(s) y_d$$

- If $H(s)$ is a rational transfer function
- Then $L(s)$ also is a rational transfer function
- Stability is determined by the poles of $L(s)$
- Same results as for the state space analysis

Control of a 1st order system

- Simplest dynamics, 1st order system
- Simple feedback works just fine
  - Static output feedback is sometimes called P control
  - P = ‘proportional’
  - The name ‘P’ is used in process industries and in servosystems, less in flight control
- Closed-loop dynamics are very well understood
- Can be used as a design template for more complex systems, cascade loops

Control of a 1^{st} order system

- First order system, integrator dynamics

$$\begin{aligned} \frac{dx}{dt} &= 0 \cdot x + bu \\ y &= 1 \cdot x \end{aligned}$$

$$A = 0 \quad B = b \quad C = 1$$

- P Control

$$u = -k(y - y_d)$$

- Closed loop dynamics

$$\frac{dx}{dt} = -kb(y - y_d)$$

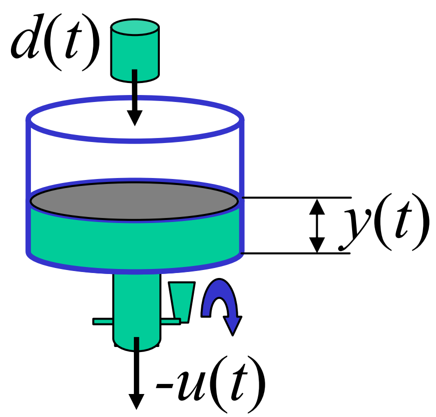
- Eigenvalue=pole

$$\lambda = -kb$$

P control - example

- Integrator plant:

$$\dot{y} = bu + w$$



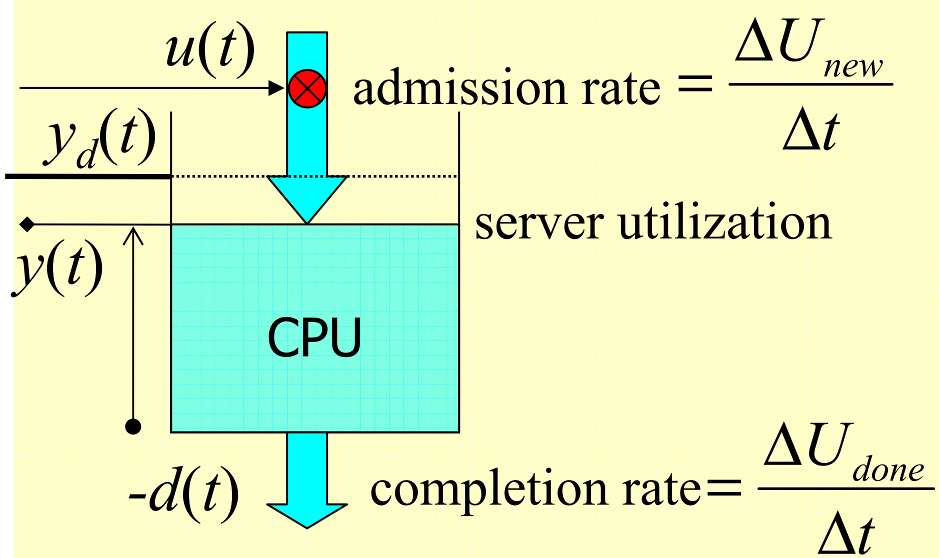
- P controller:

$$u = -k_P(y - y_d)$$

Example:
Utilization control in a video server

Video stream i

- processing time $c[i]$, period $p[i]$
- CPU utilization: $U[i]=c[i]/p[i]$



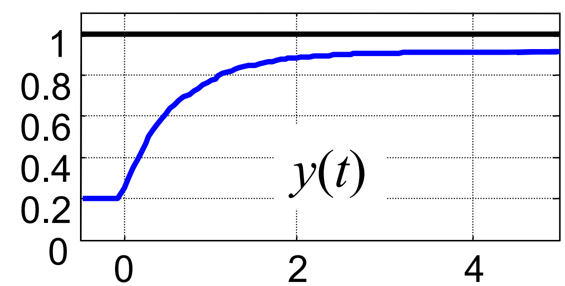
P control

- Closed-loop dynamics

$$\dot{y} + bk_P y = bk_P y_d + w \qquad y = \frac{bk_P}{s + bk_P} y_d + \frac{1}{s + bk_P} w$$

- Steady-state ($s = 0$) $\quad y_{SS} = y_d + \frac{1}{bk_P} w_{SS}$
- Step response: $T = 1/(bk_P)$

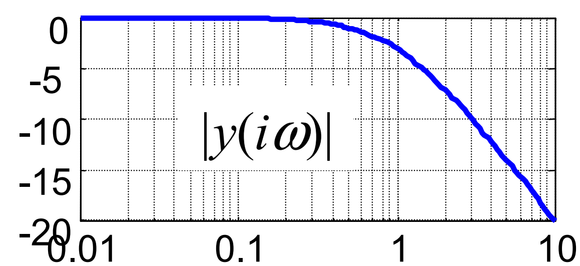
$$y(t) = y(0)e^{-t/T} + \left(y_d + \frac{1}{bk_P} w_{SS} \right) \cdot \left(1 - e^{-t/T}\right)$$



- Frequency response (bandwidth=$2\pi/T$)

$$y_d(t) = \hat{y}_d(i\omega)e^{i\omega t}$$
$$d(t) = \hat{d}(i\omega)e^{i\omega t}$$

$$|\hat{y}(i\omega)| = \frac{|\hat{y}_d(i\omega) + \hat{w}(i\omega)/(bk_P)|}{\sqrt{\omega^2/(bk_P)^2 + 1}}$$



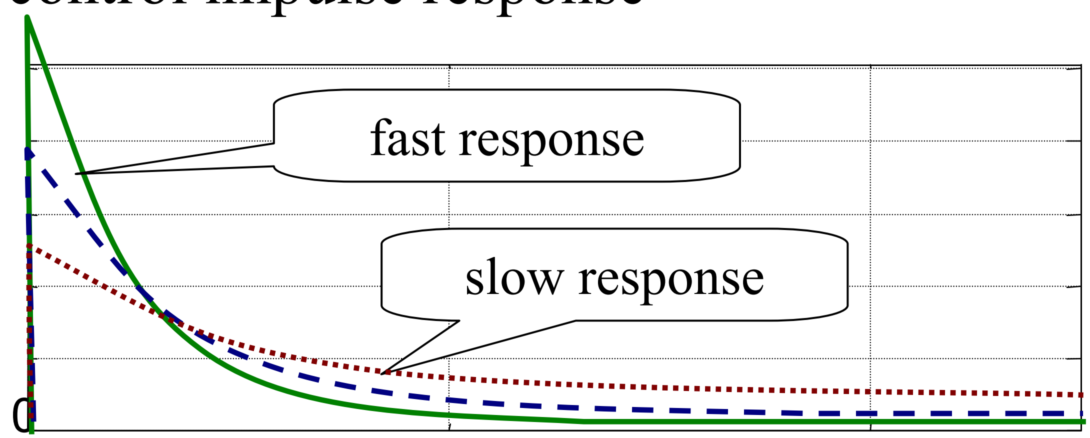
Control and Error Peaking

- Fast poles are not necessarily good
- This might mean large peak resonse
- Example: P control of an integrator

$y = h * y_d$ - closed-loop impulse response

$u = k_P h * y_d$ - control impulse response

$$h(t) = bk_P \exp(-bk_P t)$$



- Engineering design is a series of tradeoffs

I control

- 0^{th} order (feedthrough) system

$$y = d \cdot u + w,$$

- Introduce integrator into control

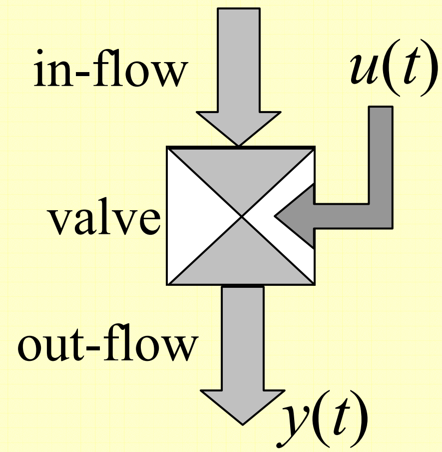
$$\dot{u} = v,$$
$$v = -k_I (y - y_d)$$

- Closed-loop dynamics

$$y = \frac{dk_I}{s + dk_I} y_d + \frac{s}{s + dk_I} w$$

Example:

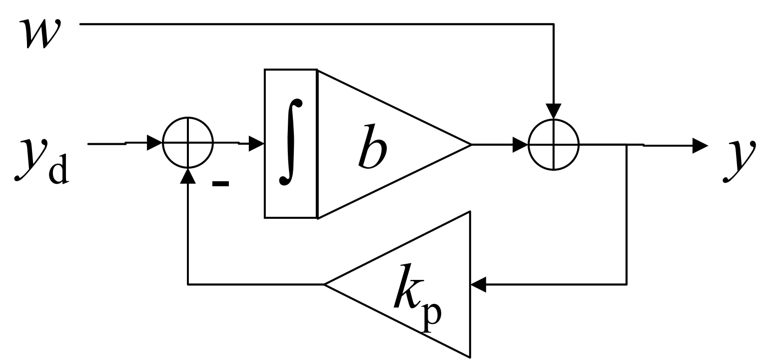
- flow through a valve



- Valves:
  - Mechanical: fluid or gas
  - Electrical: power
  - Computing: tasks
  - Comm: packets

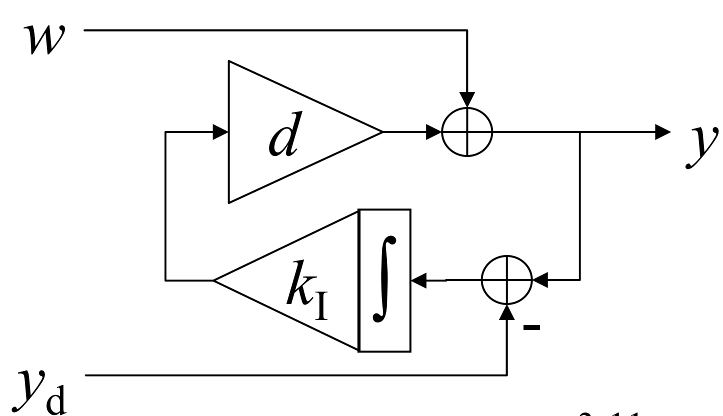
P and I control

- P control of an integrator



$$u = -k_P(y - y_d)$$
$$\dot{y} = bu + w$$

- I control of a 0th order system. Basically, the same feedback loop



$$\dot{u} = -k_I(y - y_d)$$
$$y = d \cdot u + w,$$

First order estimation - differentiator

- Differentiating filter
- Velocity estimation: $y \rightarrow v$

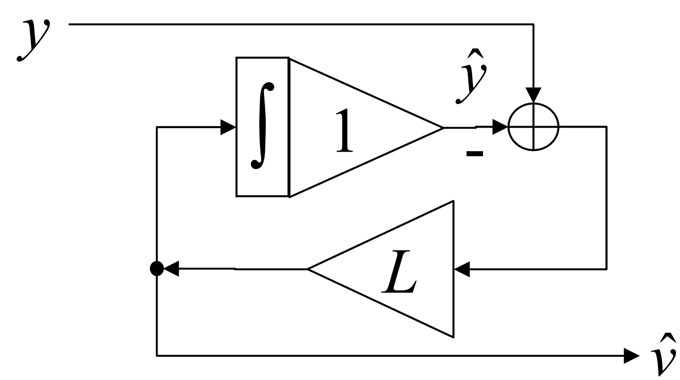
Model: $$\frac{dx}{dt} = v$$ $$y = x$$

- Observer $$\frac{d\hat{x}}{dt} = L(y - \hat{y})$$ $$\hat{y} = \hat{x}$$

$$\hat{v} = \frac{d\hat{x}}{dt} = L(y - \hat{y})$$

- Velocity estimation filter

$$\hat{v} = \frac{s}{1 + L^{-1}s} x$$



First order estimation – example

Input

Signal

Feedback

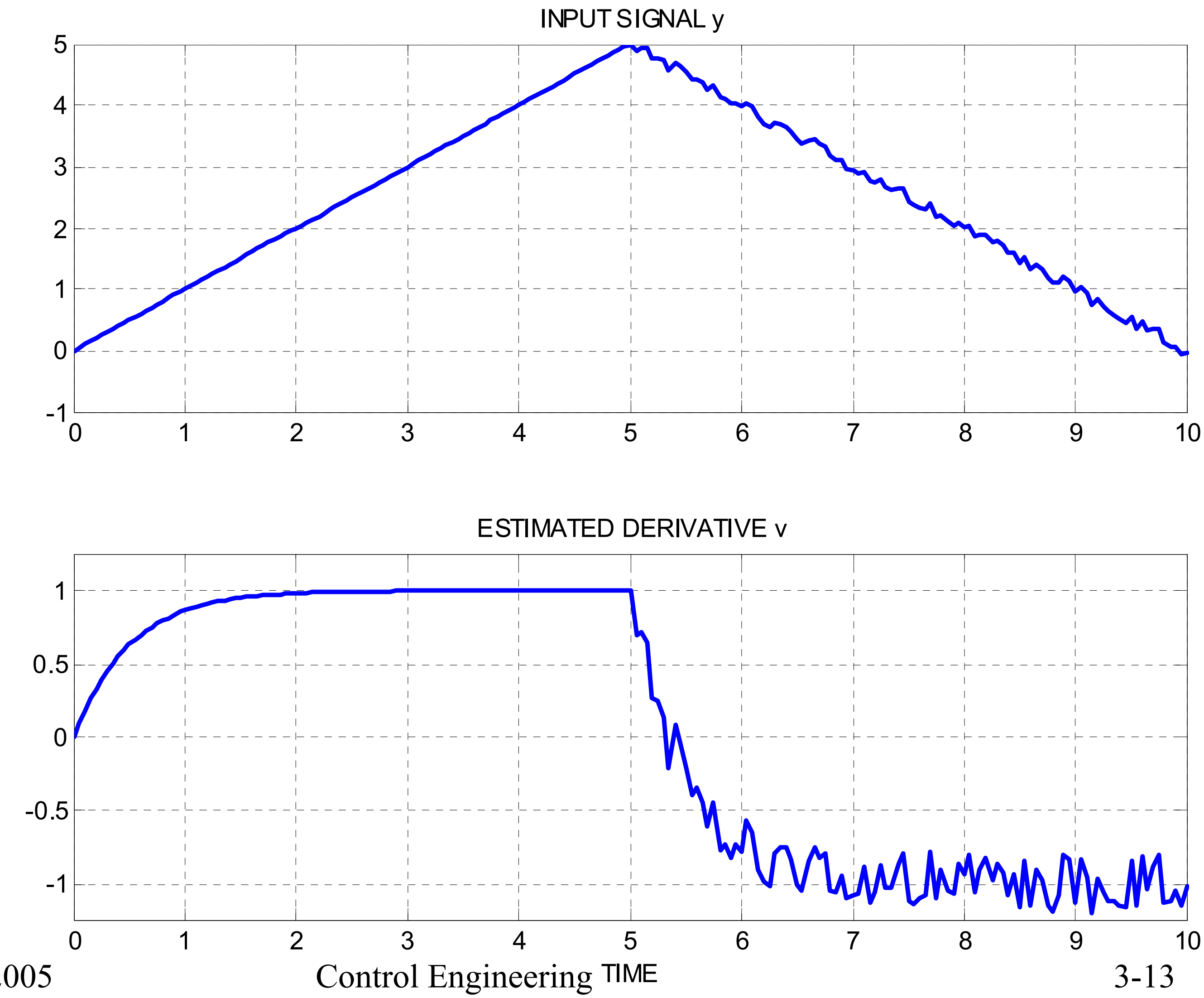
gain

$L = 2$

Output

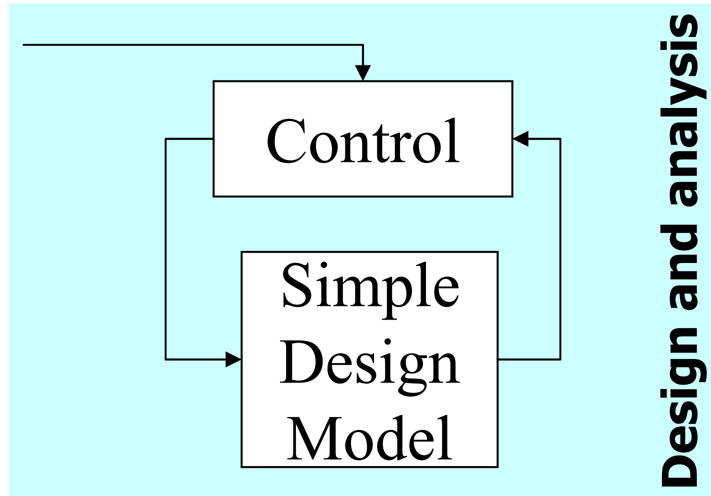
Signal

$$\hat{v} = \frac{s}{1 + L^{-1}s}\, y$$



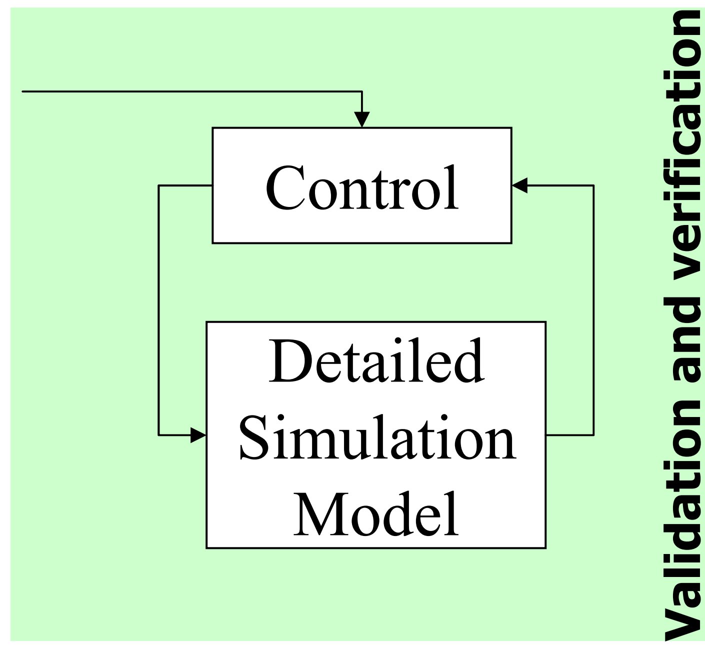
Simplified design and analysis

- Simple design model
  - Often 0^{th} or 1^{st} or 2^{nd} order model
  - Will consider typical examples
  - Use cascade loops



- Approximate model, robustness

- Analyze using a more detailed linear model
- Validate through simulation,



Example

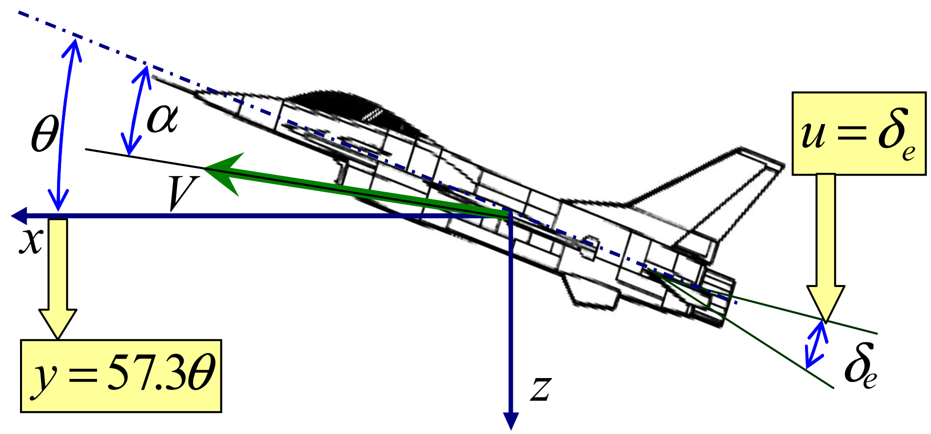
- F16 longitudinal model, constant velocity
- Assume that the velocity is maintained by regulating thrust

$$\dot{V} = 0$$

$$\dot{\alpha} = -1.02\alpha + 0.91q - 2.15 \cdot 10^{-3}\delta_e$$

$$\dot{\theta} = q$$

$$\dot{q} = 0.82\alpha - 1.08q - 0.18\delta_e$$



$$\frac{dx}{dt} = \underbrace{\begin{bmatrix} -1.02 & 0 & 0.91 \\ 0 & 0 & 1 \\ 0.82 & 0 & -1.08 \end{bmatrix}}_{A} \cdot x + \underbrace{\begin{bmatrix} -2.15 \cdot 10^{-3} \\ 0 \\ -0.18 \end{bmatrix}}_{B} \cdot u$$

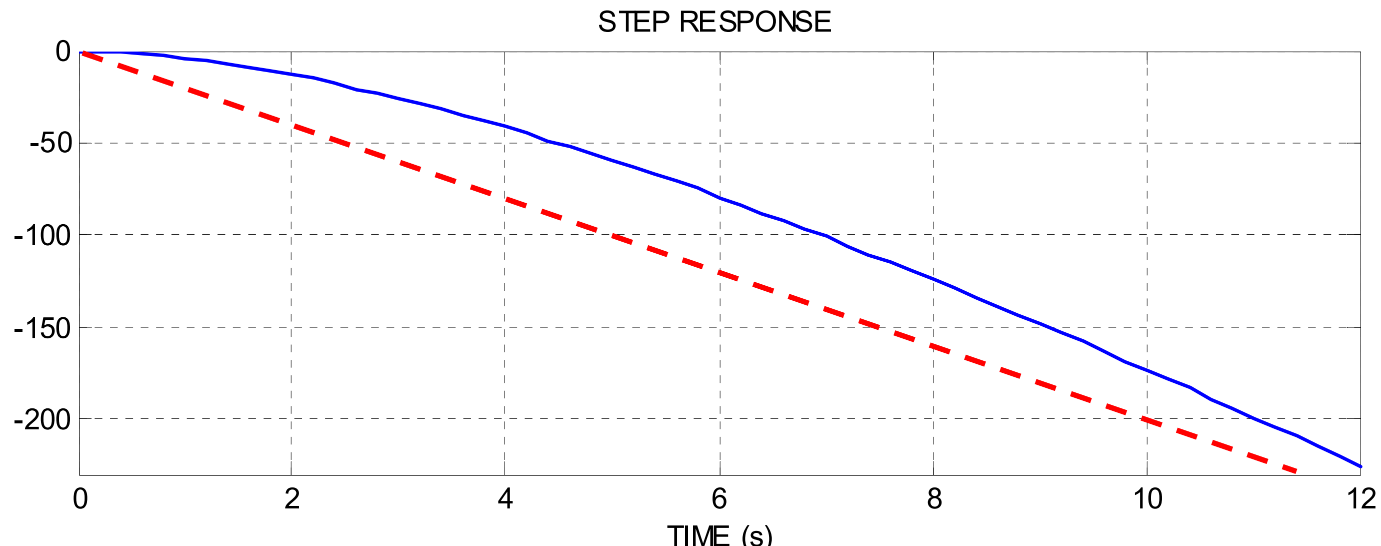
$$y = \underbrace{\begin{bmatrix} 0 & 57.3 & 0 \end{bmatrix}}_{C} \cdot x$$

```
>> eig(A)
ans =
     0
   -0.1880
   -1.9120
```

F16 Attitude Control

- Simulated step response
- At slow time scale, the integrator dynamics are dominant
- Approximate by a simple integrator model

$$\frac{d\theta}{dt} = -20\delta_e$$

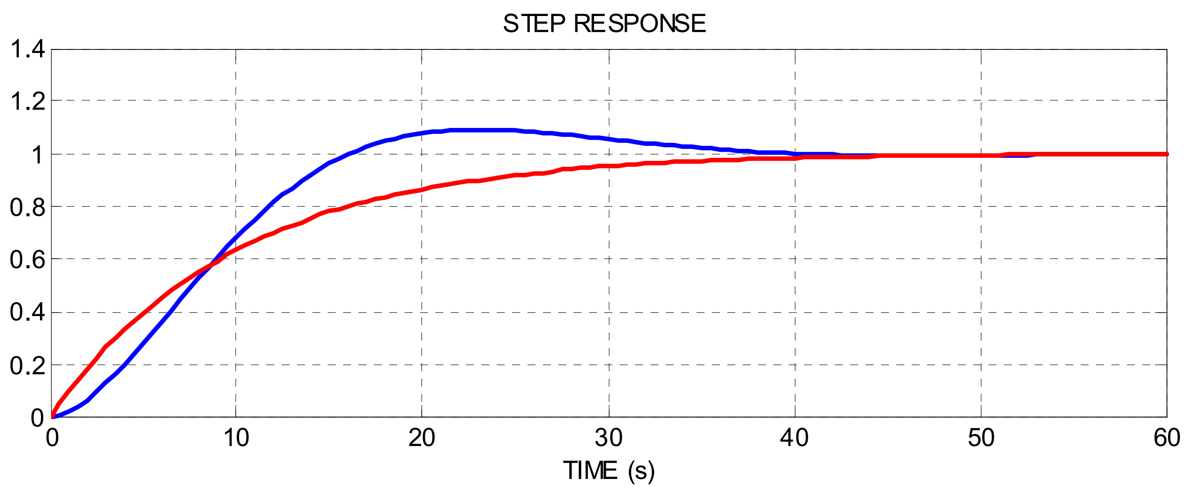


F16 Attitude Control

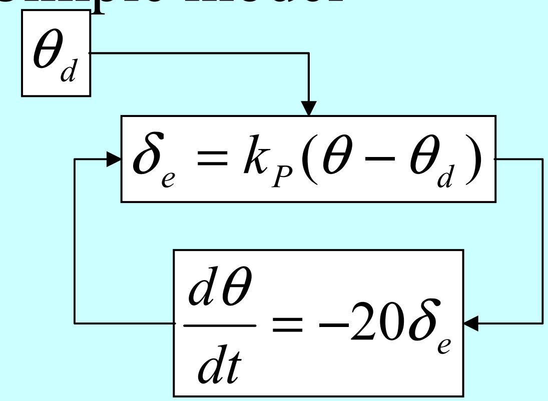
- P control design for an integrator

$$k_P = 0.005 \quad \Leftrightarrow \quad T = 1/(20k_P) = 10$$

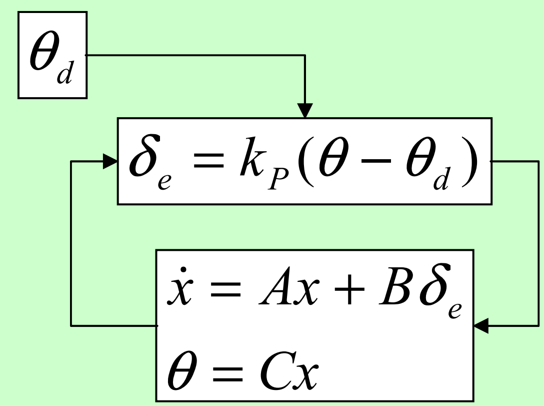
- Time responses for the simple model and for the 'detailed' model



Simple model



'Detailed' model



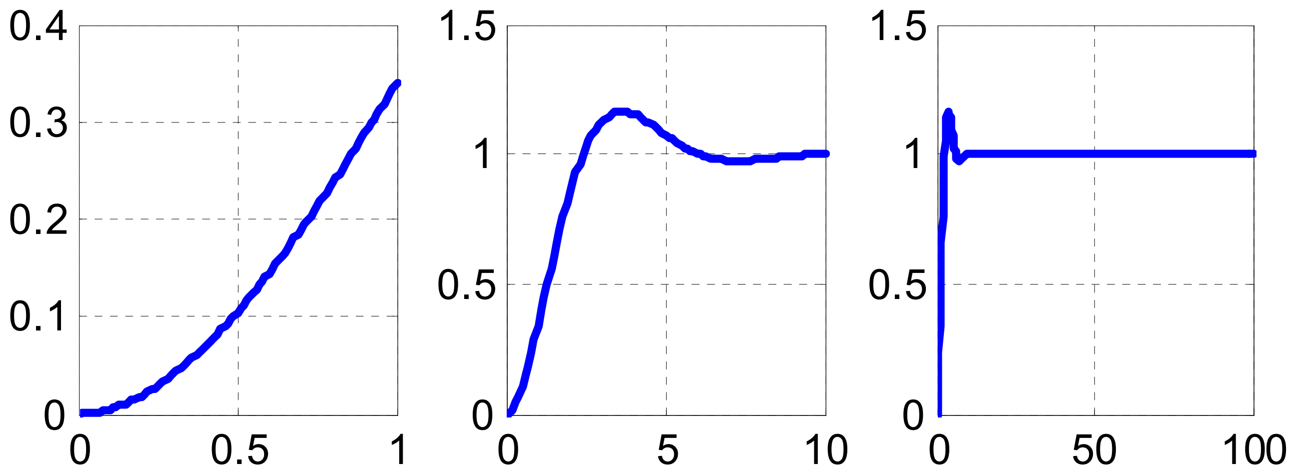
Time scale

- The same plot at different scales
- Bandwidth ←→ 1/Timescale

- Simple 2nd order model example: $H(s) = 1/(1+s+s^2)$

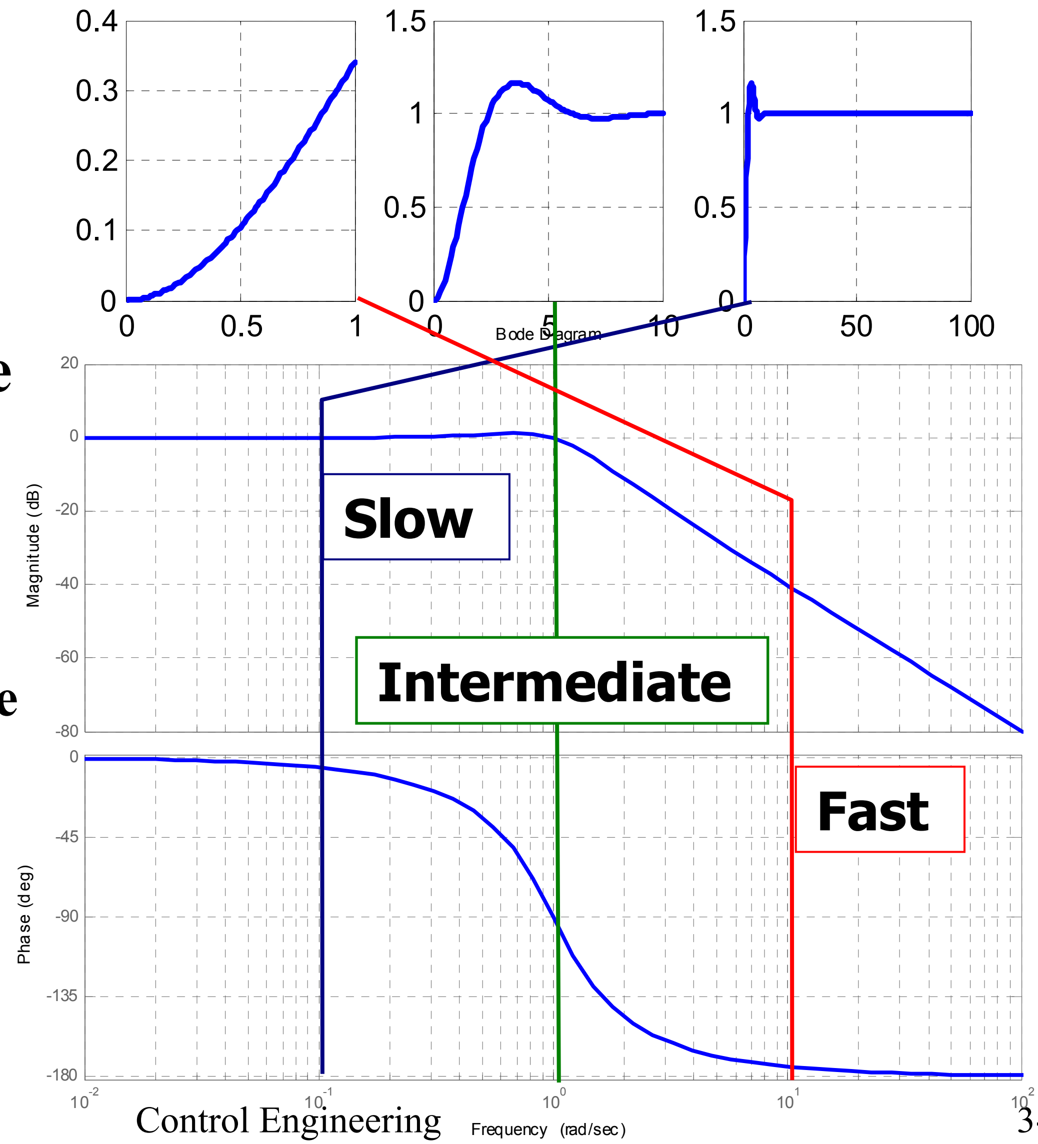
Time scales:

	Fast	**Intermediate**	**Slow**
	$H \approx 1/s^2$	$H = 1/(1+s+s^2)$	$H \approx 1$



Time Scale and Frequency Response

Time scales:



Frequency response for the example:

$H(s) = 1/(1+s+s^2)$

Bandwidth=1/Timescale

- The bandwidth is limited by model uncertainty: Lectures 9-10

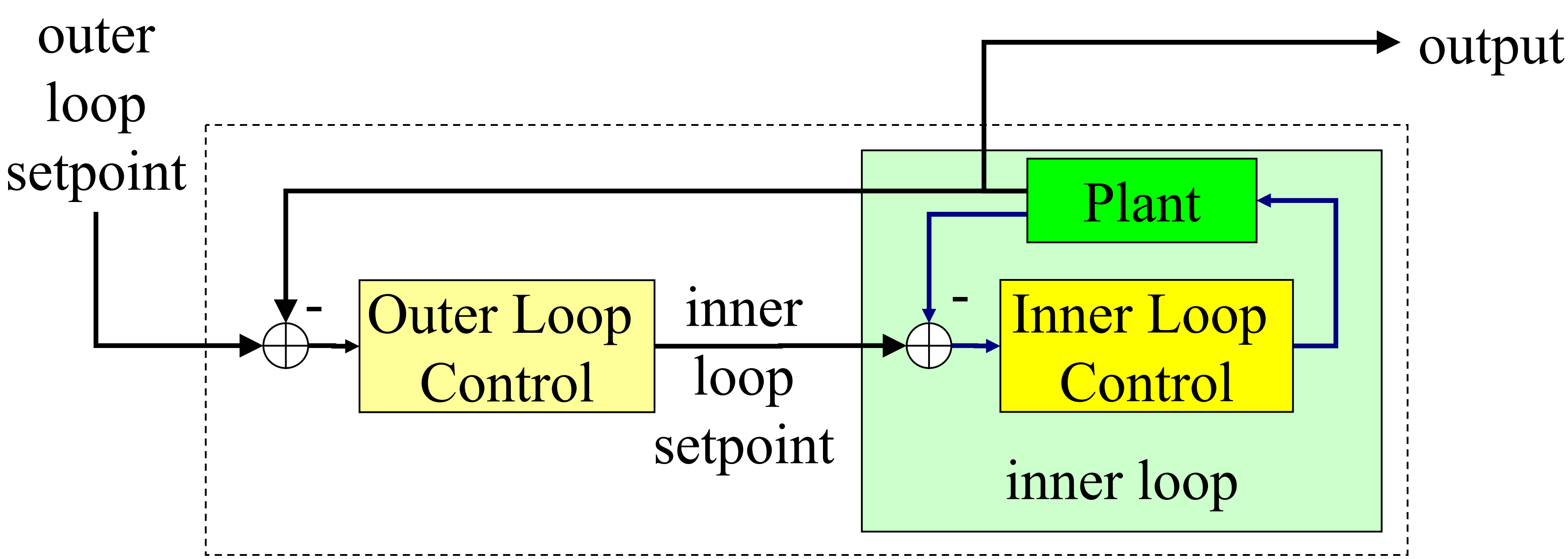
Feedback loop time scale

- Slow feedback loop
  - I control
  - Plant as a feedthrough

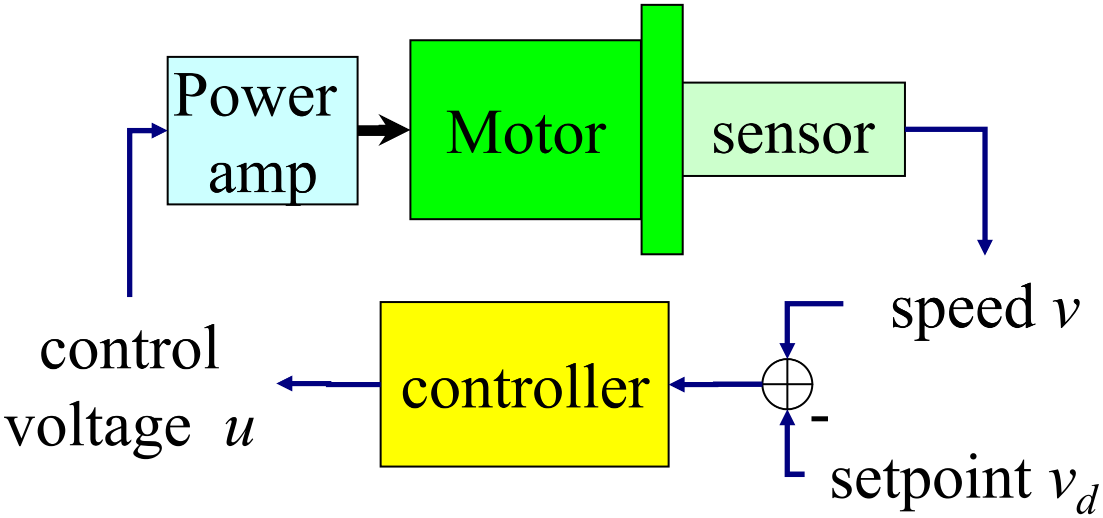
- Fast feedback loop
  - P control, plant as an integrator
  - PD control, plant as a double integrator

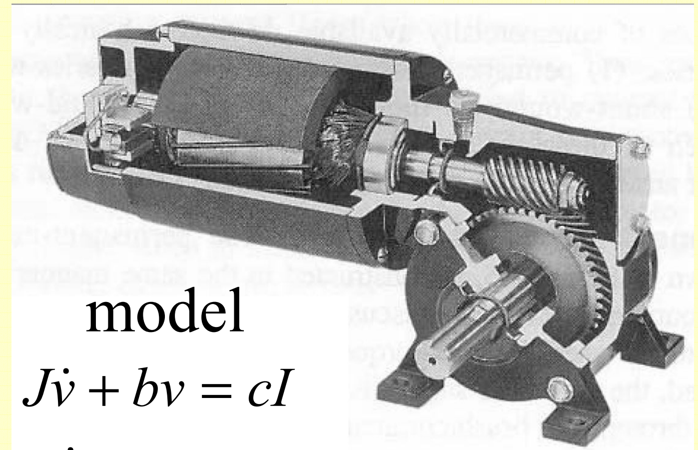
Cascaded loop design

- Inner loop has faster time scale than outer loop
- In the outer loop time scale, consider the inner loop as a 0^{th} or 1^{st} order system that follows its setpoint input



Servomotor Speed Control Example





model

$$J\dot{v} + bv = cI$$

$$L\dot{I} + RI = du$$

$$y = v$$

Transfer function

$$y = \frac{G}{(1+T_J s)(1+T_I s)} u$$

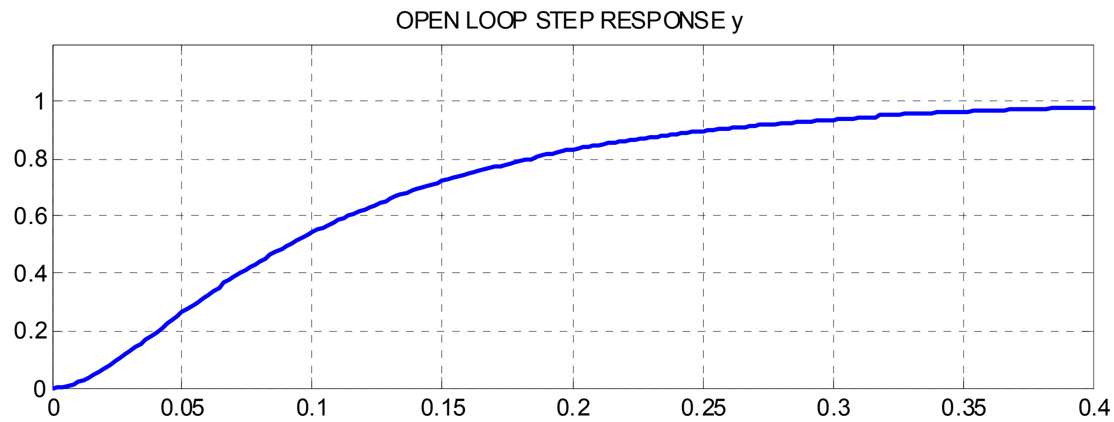
$T_J = 0.1\,\text{sec}, \qquad T_I = 0.02\,\text{sec}$

$G = 1$

- The control goal is to track a velocity setpoint
- Mechanical time constant T_J is dominant.
- Use simple model $y = \frac{1}{s} \cdot \frac{G}{T_J} u$

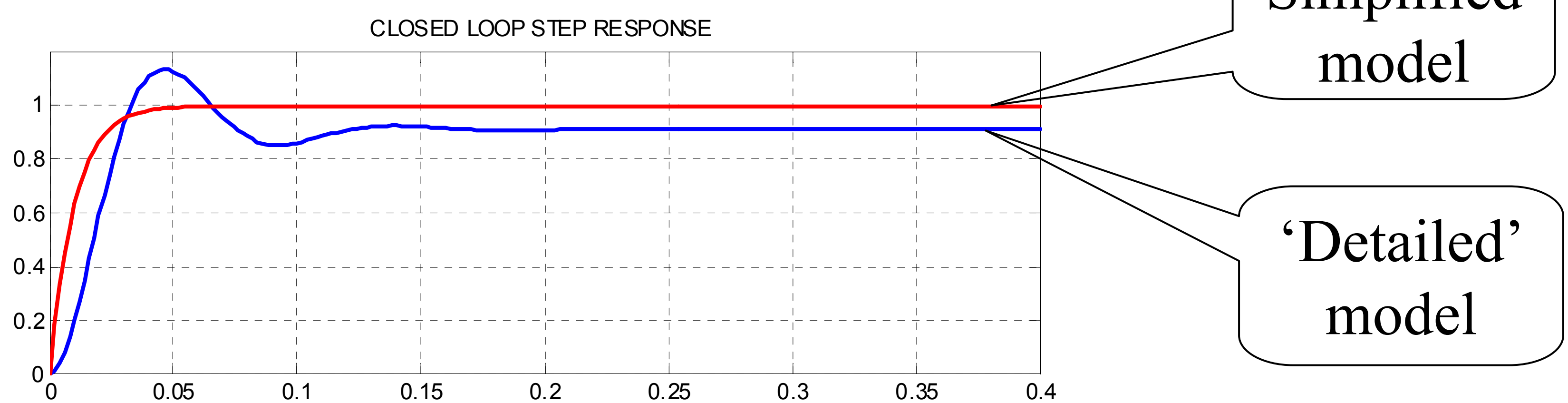
Servomotor Example, cont'd

- Design P control $u = -k_V(v - v_d)$ for the simple model $v = \frac{1}{s} \cdot \frac{G}{T_J} u$

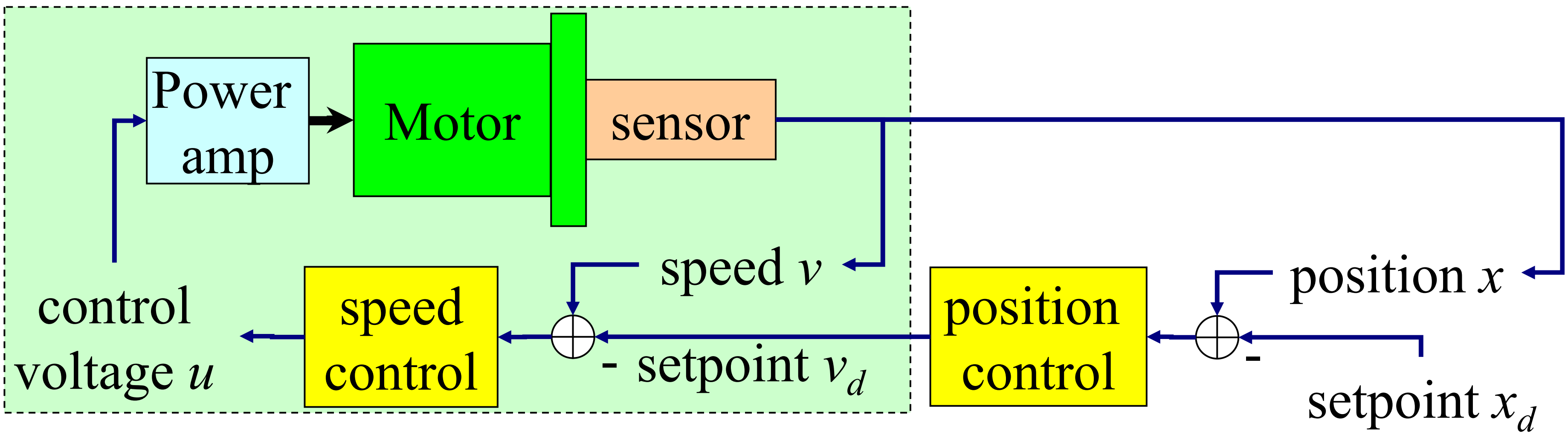


$$k_V = 10$$

$$T_{loop} = \frac{1}{k_P \cdot G / T_J} = 0.01$$



Servomotor Position Control



- Cascaded with the speed control loop
- The control goal is to track the position setpoint
- Speed loop integrator yields the dominant dynamics

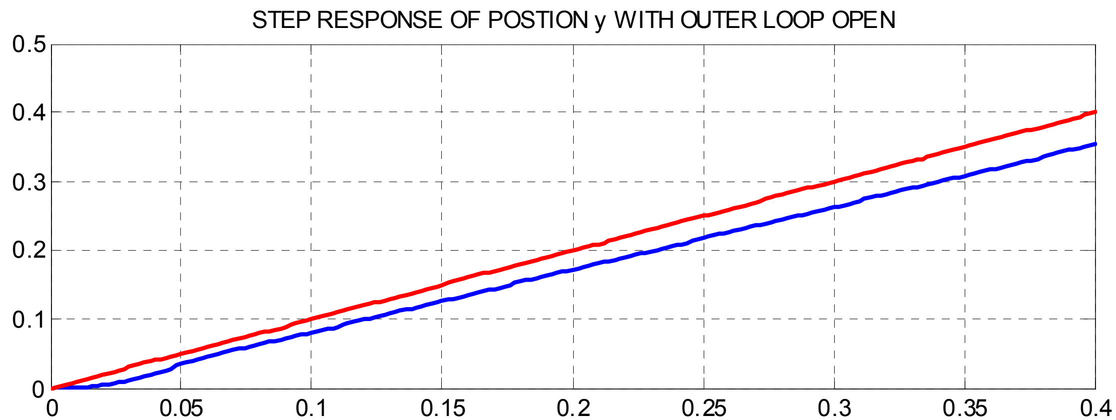
- Use simple model of the plant (inner loop)

$$\frac{dx}{dt} = v_d$$

$$y = \frac{1}{s} \cdot u$$

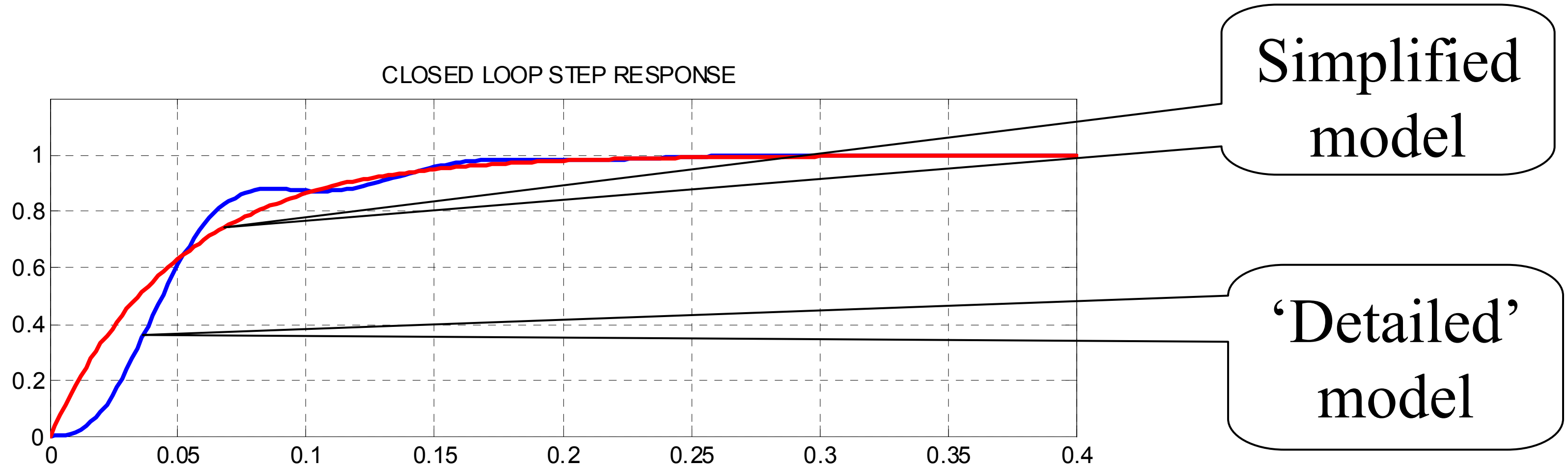
Servomotor Position, cont'd

- Design P control $v_d = -k_P(y - y_d)$ for the simple model $y = \frac{1}{s} \cdot v_d$



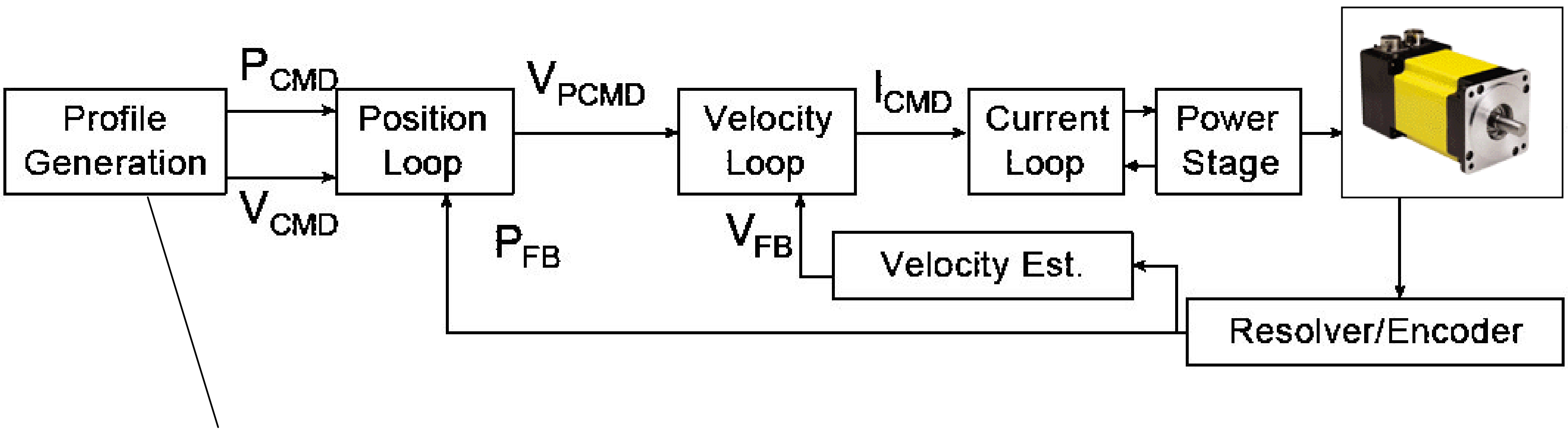
$$k_P = 20$$

$$T_{loop} = \frac{1}{1 \cdot k_P} = 0.05$$



Electric Motor Servo

- Broadly available products
- 2-4 cascaded loops depending on the sensor hardware, motor hardware, the application, and the required control functions



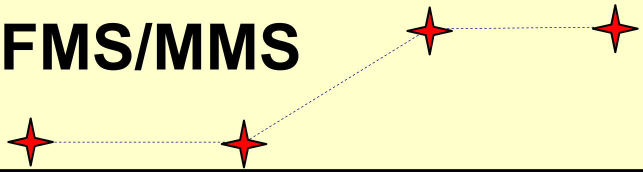
Lecture 6

Aircraft Cascaded Loops

- In practice, multivarable control design might be done for attitude control
- Otherwise, aircraft is represented as a chain of several integrators

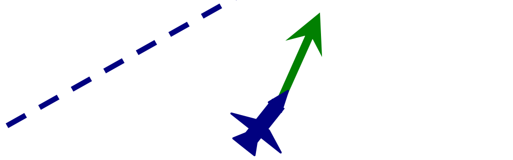
Basic cascaded loops in aircraft

FMS/MMS



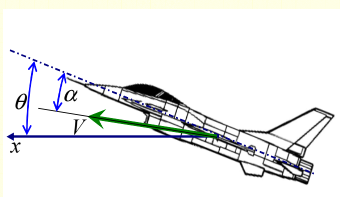
- Waypoint
- Altitude, coordinates

Guidance and Autopilot



- Translational position
- Commanded airspeed
- Translational velocity
- Attitude command

Flight Control



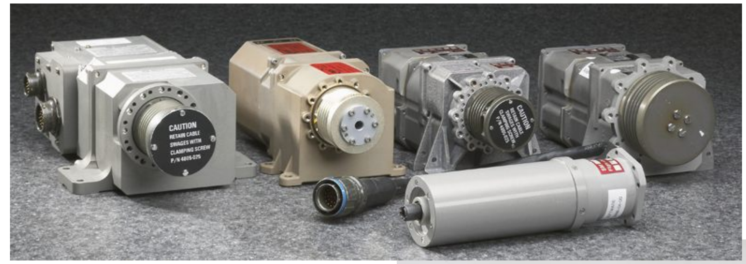
- Angular position; Attitude
- Angular rate command
- Angular rate
- Elevator position

Flight Actuators

- Actuators ⇄ Actuator servos

Aircraft Cascaded Loops

Embeded servo avionics



- Actuators, 100 hz bandwidth

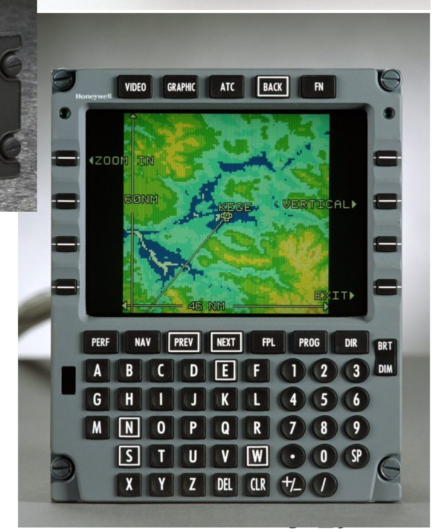
Flight Control box

- Angular rate, 2Hz bandwidth
- Angular position, 0.5Hz bandwidth

Autopilot/Guidance

- Translational velocity, 5 sec
- Translational position, 30 sec

FMS - Flight Management System



- Waypoint, 100-1000 sec

Cascaded Loop Example

- Descent/Abort Guidance
- Dale Enns, Honeywell, 1989/1997